Binomialkoeffizienten

Definitionen: Für $n, k \in \mathbb{N}_0$ gilt:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} = \frac{n(n-1)\ldots[n-(k-1)]}{k!} \qquad \text{für } 0 \le k \le n$$

$$\binom{n}{k} = 0 \qquad \text{für } k > n$$

Binomialsatz

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k \qquad (n \in \mathbb{N}_0)$$

Relationen zwischen Binomialkoeffizienten

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

Spatp.

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}.$$

Stirlingsche Formel, gültig für $n \gg 1$:

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n} \quad \Leftrightarrow \quad \ln(n!) \approx \left(n + \tfrac{1}{2}\right) \ln n - n + \tfrac{1}{2} \ln(2\pi)$$

~~Spezielle Werte der trigonometrischen Funktionen~~

ϕ [°]	0°	30°	45°	60°	90°	120°	135°	150°	180°
ϕ [rad]	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$	$\frac{2}{3}\pi$	$\frac{3}{4}\pi$	$\frac{5}{6}\pi$	π
$\sin\phi$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\phi$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\phi$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot\phi$	$\mp\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\mp\infty$

ϕ [°]	180°	210°	225°	240°	270°	300°	315°	330°	360°
ϕ [rad]	π	$\frac{7}{6}\pi$	$\frac{5}{4}\pi$	$\frac{4}{3}\pi$	$\frac{3}{2}\pi$	$\frac{5}{3}\pi$	$\frac{7}{4}\pi$	$\frac{11}{6}\pi$	2π
$\sin\phi$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\phi$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\phi$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\cot\phi$	$\mp\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\mp\infty$

$\cot(0°) = \mp\infty$ bedeutet: $\lim\limits_{\phi\to 0-} \cot\phi = -\infty$, $\lim\limits_{\phi\to 0+} \cot\phi = +\infty$.

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sin(\alpha \pm \beta) = \sin\alpha\,\cos\beta \pm \cos\alpha\,\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\,\cos\beta \mp \sin\alpha\,\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\,\tan\beta}$$

$$\cot(\alpha \pm \beta) = \frac{\cot\alpha\,\cot\beta \mp 1}{\cot\beta \pm \cot\alpha}$$

$$\sin(2\alpha) = 2\sin\alpha\,\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

Geometrische Summe

Es sei $q \neq 0$. Für $q \in \mathbb{C}$ und $n \in \mathbb{N}_0$ gilt:

$$\sum_{\nu=0}^{n} q^\nu = \begin{cases} \dfrac{q^{n+1}-1}{q-1}, & q \neq 1 \\ (n+1), & q = 1 \end{cases}$$

Mit $\varphi_1 = \arctan\left|\frac{y}{x}\right|$ ist

φ	$x < 0$	$x \geq 0$
$y \geq 0$	$\pi - \varphi_1$	φ_1
$y < 0$	$\pi + \varphi_1$	$2\pi - \varphi_1$

Moivre:

$$z^n = |z|^n\,(\cos n\varphi + i\,\sin n\varphi)$$

$$^b\!\log x = {}^b\!\log a \cdot {}^a\!\log x$$

$$^b\!\log a = \frac{1}{^a\!\log b}$$

$2^{10} = 1024$; $e^3 \approx 20$; $\sqrt{e} \approx 1{,}65$

$$(1+x)^\mu = 1 + \binom{\mu}{1}x + \binom{\mu}{2}x^2 + \ldots = \sum_{n=0}^{\infty}\binom{\mu}{n}x^n \qquad (|x|<1,\ \mu \in \mathbb{R})$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \sum_{n=0}^{\infty} x^n \qquad (|x| < 1)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \pm \ldots = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} \qquad (-1 < x \leq 1)$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty}\frac{x^n}{n!} \qquad (|x| < \infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \mp \ldots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \qquad (|x| < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \mp \ldots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} \qquad (|x| < \infty)$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \ldots \qquad (|x| < \tfrac{\pi}{2})$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C \qquad -1 \neq r \in \mathbb{R}$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x\,dx = e^x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \frac{dx}{\cos^2 x} = \tan x + C$$

$$\int \frac{dx}{\sin^2 x} = -\cot x + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{dx}{1+x^2} = \arctan x + C$$

2. $\int R(\sin x, \cos x)\,dx$

$R(u, v)$ sei eine bezüglich u und v rationale Funktion

Substitution: $t = \tan\frac{x}{2}$

$$\rightarrow \int R(\sin x, \cos x)\,dx = \underbrace{\int R\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)\frac{2}{1+t^2}}_{\text{rational in } t}\,dt$$

3. $\int R(\sin^2 x, \cos^2 x)\,dx$

Substitution: $y = \tan x$

$$\rightarrow \int R(\sin^2 x, \cos^2 x)\,dx = \underbrace{\int R\left(\frac{y^2}{1+y^2}, \frac{1}{1+y^2}\right)\frac{1}{1+y^2}}_{\text{rational in } y}\,dy$$

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

Funktion $\Gamma(x)$ (Gammafunktion) als das Integral

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt \qquad x > 0$$

$$\Gamma(n+1) = n! \qquad n \in \mathbb{N}_0$$

$\Gamma(x+1) = x\Gamma(x)$ und der spezielle Wert $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$\Gamma(x)$ nicht für BC/MM/LA

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$\cos\phi$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
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$\cot\phi$	$\mp\infty$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\mp\infty$

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$\cos\phi$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\phi$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\pm\infty$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
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