

# Vorkurs Lösungen

## Mathematik f. Chemiker

### BLOCK I

#### 1. Faktorisieren

- (1)  $(a - b)(a + b)(a^2 + b^2)$       (6)  $(3c - 4)(3c - 8)$   
(2)  $(x - 3)(x + 3)(x^2 + 9)$       (7)  $(x - y)(x^2 + xy + y^2)$   
(3)  $(\frac{c}{3} - \frac{x}{2})(\frac{c}{3} + \frac{x}{2})(\frac{c^2}{9} + \frac{x^2}{4})$       (8)  $(2b + 5)(4b^2 - 10b + 25)$   
(4)  $(n + 8)(n - 2)$       (9)  $(3x + 5)(x^2 + 1)$   
(5)  $(x - 3)(x + 2)$       (10)  $(x + 1)(x^3 + 2) =$   
 $(x + 1)(x + \sqrt[3]{2})(x^2 - \sqrt[3]{2}x + \sqrt[3]{4})$

#### 2. Polynomdivision

- (11)  $2a - 3b$       (12)  $3x + 8z$   
(13)  $4a^2 - 2b^2$   
(14)  $(a + b)(a^2 + b^2) = a^3 + a^2b + ab^2 + b^3$   
(15)  $27x^6 - 18x^4z + 12x^2z^2 - 8z^3$       (16)  $1 + x + 2y$   
(17)  $6 - 7a$       (18)  $-x^3 + 2x^2 - 3x + 1$   
(19)  $\frac{1}{x^{16}(x - y)^2}$       (20)  $x^{16}(x - y)(x + y)^2$

#### 3. Gleichungen

- (21)  $x = -5$       (22)  $x = 1$   
(23)  $x = 11$       (24)  $x = a + n$   
(25)  $x = 3$

## BLOCK II

### 1. Verhältnisgleichungen

- |           |            |
|-----------|------------|
| (a) 3 : 5 | (f) 3 : 5  |
| (b) f     | (g) 3 : 8  |
| (c) 1 : 4 | (h) 4 : 9  |
| (d) f     | (i) 7 : 10 |
| (e) f     | (j) f      |

### 2. Gewichtsprozent

- (a)  $\text{CO}_2$      $\text{C} : \text{O} = 12\text{g} : 32\text{g} = 3\text{g} : 8\text{g}$   
 $\frac{3}{3+8} \times 100 = 27.2727 \Rightarrow 27,3 \text{ Gew.-% C}; 72,7 \text{ Gew.-% O}$
- (b)  $\text{H}_2\text{SO}_4$      $\text{H} : \text{S} : \text{O} = 1\text{g} : 16\text{g} : 32\text{g}$   
2 Gew.-% H; 32,7 Gew.-% S; 65,3 Gew.-% O
- (c)  $\text{H} : \text{N} : \text{O} = 1\text{g} : 14\text{g} : 48\text{g}$   
1,6 Gew.-% H; 22,2 Gew.-% N; 76,2 Gew.-% O
- (d)  $\text{H} : \text{Br} = 1\text{g} : 80\text{g}$  1,2 Gew.-% H; 98,8 Gew.-% Br
- (e)  $\text{C} : \text{H} : \text{O} = 36\text{g} : 5\text{g} : 48\text{g}$   
40,4 Gew.-% C; 5,6 Gew.-% H; 53,9 Gew.-% O
- (f)  $\text{C} : \text{H} : \text{O} = 6\text{g} : 1\text{g} : 8\text{g}$   
40 Gew.-% C; 6,7 Gew.-% H; 53,3 Gew.-% O
- (g)  $\text{C} : \text{H} : \text{Cl} = 144\text{g} : 10\text{g} : 71\text{g}$   
64 Gew.-% C; 4,4 Gew.-% H; 31,6 Gew.-% Cl
- (h)  $\text{C} : \text{H} : \text{Cl} : \text{N} : \text{O} = 144\text{g} : 8\text{g} : 71\text{g} : 28\text{g} : 32\text{g}$   
50,9 Gew.-% C; 2,8 Gew.-% H; 25,1 Gew.-% Cl;  
9,9 Gew.-% N; 11,3 Gew.-% O
- (i)  $\text{Si} : \text{F} = 7\text{g} : 19\text{g}$  26,9 Gew.-% Si; 73,1 Gew.-% F
- (j)  $\text{C} : \text{H} : \text{Cl} = 24\text{g} : 2\text{g} : 213\text{g}$   
10 Gew.-% C; 0,8 Gew.-% H; 89,2 Gew.-% Cl

3. (a)

$$\begin{array}{rcl}
 1 \text{ mol Fe} & = & 55,8 \text{ g} \\
 1 \text{ mol O} & = & 16 \text{ g} \\
 189,88 \text{ kg} & \text{sind} & \frac{189,88 \text{ kg}}{55,8} = 3403 \text{ mol Fe} \\
 81,6 \text{ kg} & \text{sind} & \frac{81,6 \text{ kg}}{16} = 5100 \text{ mol Fe} \\
 \text{Insgesamt } 3403 + 5100 & = & 8503 \\
 \text{Fe: } 3403/8503 = 0,4 & \rightarrow & 40 \text{ Mol \% Fe} \\
 \text{O: } 5100/8503 = 0,6 & & 60 \text{ Mol \% O}
 \end{array}$$

⇒ Formel: 40:60=2:3 ⇒ Fe<sub>2</sub>O<sub>3</sub>

- (b) 28,6 Mol % P; 71,4 Mol % O      P<sub>2</sub>O<sub>5</sub>  
 (c) 27,3 Mol % C ; 54,5 Mol % H ; 18,2 Mol % O      C<sub>3</sub>H<sub>6</sub>O<sub>2</sub>  
 (d) 16,7 Mol % K ; 16,7 Mol % Cl ; 66,4 Mol % O      KClO<sub>4</sub>  
 (e) 25 Mol % C ; 62,5 Mol % H ; 12,5 Mol % I      C<sub>2</sub>H<sub>5</sub>I

4. (a) 2 Cu + O<sub>2</sub> → 2 CuO

2 mol Cu + 1 mol O<sub>2</sub> liefern 2 mol CuO

2 · 63,55g Cu + 32g O<sub>2</sub> liefern 159.1g CuO

Verhältnisgleichung:

$$\frac{37}{159,1} = \frac{X_{\text{Cu}}}{2 \cdot 63,55} \quad \rightarrow \quad X_{\text{Cu}} = 29.56 \text{ g Cu} \\
 X_{\text{O}} = 7.44 \text{ g O}_2$$

- (b) 3.5g H<sub>2</sub> ; 123.5g Cl<sub>2</sub>  
 (c) 67g CH<sub>3</sub>COOC<sub>2</sub>H<sub>5</sub> ; 13.7g H<sub>2</sub>O  
 (d) 7.3g HCHO ; 14.1g CH<sub>3</sub>-CO-CH<sub>3</sub>  
 (e) 23.5g Al ; 92.5g Cl<sub>2</sub>

zu (d) 1mol CH<sub>2</sub>O (30g) + 1mol C<sub>3</sub>H<sub>6</sub>O (58g) liefern 1mol C<sub>4</sub>H<sub>6</sub>O (70g)

Verhältnisgleichung:

$$\frac{17}{70} = \frac{X_{\text{CH}_2\text{O}}}{30} \quad \rightarrow \quad X_{\text{CH}_2\text{O}} = 7.29 \text{ g} \\
 \text{analog} \quad X_{\text{C}_3\text{H}_6\text{O}} = 14.09 \text{ g}$$

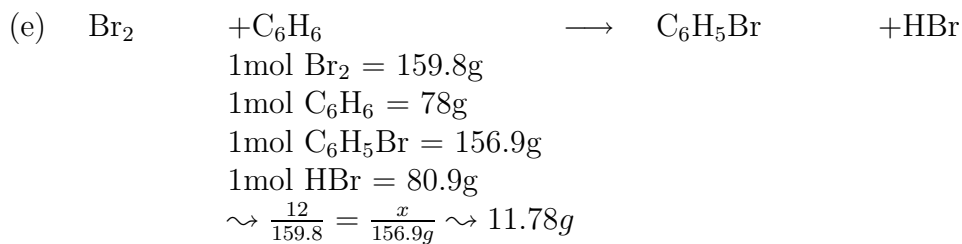
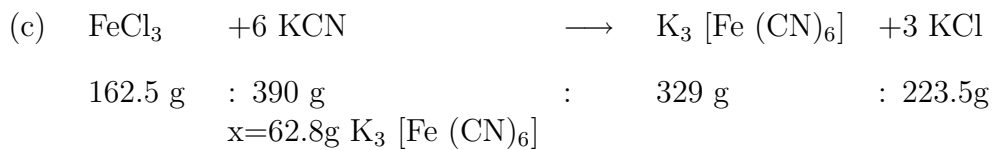
5. (a) Verhältnisse:

$$1 \text{ mol H}_2 = 2 \text{ g}$$

$$1 \text{ mol N}_2 = 28 \text{ g} \quad \rightsquigarrow 3 \cdot 2 : 28 : 2 \cdot 17$$

$$1 \text{ mol NH}_3 = 17 \text{ g} \quad = 3 : 14 : 17$$

$$\text{Einsatz: } 37 \text{ g H}_2 : \frac{37}{3} = \frac{x}{17 \text{ g}} \rightsquigarrow x = 209.7 \text{ g NH}_3$$



6. (a) 13g 15%ige Salzsäure enthält:

$$15 \cdot 13 \text{ g} / 100 = 1.95 \text{ g HCl}$$

$$85 \cdot 13 \text{ g} / 100 = 11.05 \text{ g H}_2\text{O}$$

$$1 \text{ mol HCl} = 36.5 \text{ g} \rightsquigarrow 1.95 \text{ g} = 0.053 \text{ mol HCl}$$

$$1 \text{ mol H}_2\text{O} = 18 \text{ g} \rightsquigarrow 11.05 \text{ g} = 0.61 \text{ mol H}_2\text{O}$$

(b)  $7.32 \text{ g} = 0.183 \text{ mol NaOH}$  ;  $724.68 \text{ g} = 40.26 \text{ mol H}_2\text{O}$ ;

(c)  $45.5 \text{ g} = 0.8125 \text{ mol Fe}$  ;  $19.5 \text{ g} = 0.609 \text{ mol S}$

(d)  $2.5 \text{ g} / 32 \frac{\text{g}}{\text{mol}} = 0.0781 \text{ mol Methanol}$  ;  $7.5 \text{ g} / 46 \frac{\text{g}}{\text{mol}} = 0.163 \text{ mol Ethanol}$

(e)  $44.64 \text{ g} / 93 \frac{\text{g}}{\text{mol}} = 0.48 \text{ mol C}_6\text{H}_5\text{NH}_2$  ;  $327.4 \text{ g} / 78 \frac{\text{g}}{\text{mol}} = 4.2 \text{ mol Benzol}$

7. (a)  $\text{HCl} + \text{CH}_3\text{CH}_2\text{OH} \rightarrow \text{H}_2\text{O} + \text{CH}_3\text{CH}_2\text{Cl}$ ; Ausbeute: 67 %  
 Einsatz: 0,5 mol HCl; 0,5 mol  $\text{C}_2\text{H}_5\text{OH}$ ;  
 1 mol HCl = 36.5g  
 1 mol  $\text{CH}_3\text{CH}_2\text{Cl}$  = 64.5g  
 Einsatz 0.5 mol  $\leadsto$  Ausbeute 0.5 mol = 32.25g  
 $\leadsto$  reduzierte Ausbeute (67%) = 21.6g
- (b)  $2 \text{C}_2\text{H}_5\text{OH} \rightarrow \text{C}_2\text{H}_5\text{OC}_2\text{H}_5 + \text{H}_2\text{O}$ ; Ausbeute: 48 %  
 Einsatz: 1 mol  $\text{C}_2\text{H}_5\text{OH}$  = 46g  
 1 mol  $\text{C}_4\text{H}_{10}\text{O}$  = 74g  
 1mol  $\text{C}_2\text{H}_5\text{OH}$  ergibt  $\frac{1}{2}$  mol  $\text{C}_2\text{H}_5\text{OC}_2\text{H}_5$  = 37g  
 48% Ausbeute = 17.76g
- (c) 28.3g  $\text{P}_4\text{O}_{10}$
- (d) 20.5g  $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$
- (e) 83.6g  $\text{C}_4\text{H}_9\text{Br}$

## BLOCK III

### Logarithmieren, Potenzieren

#### 3.1

1. 1000
2.  $\frac{1}{2} \lg a + \frac{1}{2} \lg 3 + \frac{1}{2}$
3.  $\frac{1}{1000}$
4.  $\frac{2}{3} \ln a - \frac{1}{b} \ln c$
5. 1
6. 1
7.  $e^4$
8.  $\lg 4 + \lg \pi + 3 \lg r - \lg 3$
9. 2
10. 4
11. 57
12.  $\lg 9$
13. 5
14. 1
15.  $x(\ln 3 + \ln 5)$
16.  $5(\lg a - \lg y) + 3(\lg b - \lg x)$
17.  $\lg(q^n - 1) - \lg(q - 1) = \lg(q^{n-1} + q^{n-2} + \dots + q + 1)$
18.  $\frac{1}{5} \ln(a + b) - \frac{1}{2} \ln(a - b)$
19.  $\lg(a - b) + \lg(a^6 + a^5b + a^4b^2 + \dots + b^6)$
20.  $\lg(x^{\frac{1}{6}}) + \lg(y^{\frac{3}{4}})$
21.  $x = \frac{4}{3}$
22.  $x = 19$
23.  $x = \pm \left( \frac{\ln d}{\ln c + \ln d - 1} \right)^{\frac{1}{2}}$
24.  $2^{x+1} \left( 1 + \frac{1}{8} \right) = 3^{x-1} \left( 1 - \frac{1}{9} \right) \rightsquigarrow 2^{x+1} \frac{9}{8} = 3^{x-1} \frac{8}{9} \rightsquigarrow x = 5$
25.  $x_1 = 1 \quad x_2 = -3$
26.  $x = \frac{\ln 13}{\ln 13 - \ln 15} \approx -17.9$
27.  $\pm(\lg 2)^{1/2}$
28.  $x = y$

29.  $e^{\ln x} = 10^{\lg x} \quad | \ln$   
 $\ln x = \ln(10^{\lg x}) = \ln 10 \cdot \lg x$   
 analog  $\lg x = \ln x \cdot \lg e$   
 insbesondere  $\ln 10 = \frac{1}{\lg e}$
30.  ${}^a \log x = {}^b \log x \cdot {}^a \log b$
31.  $\lg x \left( \stackrel{29.}{=} \frac{\ln x}{\ln 10} \right) = \ln y \ln 10 \rightsquigarrow x = y^{(\ln 10)^2}$
32.  $x = y^{\lg e}$
33.  $x_1 = 2 \quad x_2 = 3$
34.  $x = 13$
35.  $x = -4$
36.  $x = 9a$
37.  $y = 4 \quad x = 5$
38.  $x = 1 \quad y = 2$
39.  $x = 2 \quad y = 1$
40. 3
41.  $a^{\frac{1}{ax}}$
42. 1
43.  $4b^{\frac{n+5}{5}}$
44.  $b^{2a-1}$
45.  $|a + b + c|$
46.  $4\sqrt{x}$
47.  $x^{-2b}$
48.  $16a^2 \sqrt[4]{b^9}$
49.  $2\sqrt{x+c+d}$
50.  $x = 5$

### 3.2 Vermischte Aufgaben

1.  $x = 5$
2.  $x = 1/4$
3.  $x = 5/2$
4.  $x = \pm 4$
5.  $x = \pm 3, x = \pm 2$
6.  $x = \pm 15$
7.  $x = \pm(a - b)$
8.  $x = \pm 13$
9.  $y = \pm 9 \quad x = \pm 6$
10.  $x_1 = \pm 1/2, y_1 = \pm \sqrt{17/3}$   
 $x_2 = \pm \frac{1}{2} \sqrt{17}, y_2 = \pm \sqrt{1/3}$
11.  $x_1 = 2, y_1 = 3 \quad x_2 = -7/5, y_2 = -7/9$

## BLOCK IV

### Differentiation

#### 4.1 Algebraische Ausdrücke

1.  $y' = 2 - 6x - 15x^2 - 23x^3 + 45x^4$

2.  $y' = -\frac{1}{x^2} - \frac{6}{x^3} - \frac{6}{x^4}$

3.  $y' = \frac{1}{\sqrt{x}} + \frac{2}{\sqrt[3]{x^2}} - 3\sqrt{x}$

4.  $y' = \frac{2x}{\sqrt[3]{9x^4}} = \frac{2}{3}\sqrt[3]{\frac{3}{x}}$

5.  $\frac{ds}{dt} = 8t(t^2 - 3)^3$

6.  $f'(x) = \frac{x + 3}{\sqrt{x^2 + 6x + 3}}$

7.  $y' = \frac{2x^3 - 13x}{\sqrt{(4 - x^2)^3}}$

8.  $y' = -\frac{1}{x^3} - \frac{30x(1 - 5x)^5 + \frac{2}{3}(1 - 5x)^6}{\sqrt[3]{x^5}}$

9.  $y = 2x(x^2 + 3)^3 (11x^3 + 9x - 20)$

10.  $y' = \frac{1}{\sqrt{x-1}} - \frac{x}{2\sqrt{(x-1)^3}}$

$$y'' = -\frac{1}{\sqrt{(x-1)^3}} + \frac{3x}{4\sqrt{(x-1)^5}}$$



$$11. \quad y' = -\frac{x}{\sqrt{(x^2 - 9)^3}}$$

$$y'' = -\frac{1}{\sqrt{(x^2 - 9)^3}} + \frac{3x^2}{\sqrt{(x^2 - 9)^5}}$$

$$12. \quad y' = \frac{-2(x^5 + 6x^4 + 8x^3 + 4x + 8)}{(x^4 - 4)^2}$$

$$13. \quad y' = -\frac{1}{2\sqrt{1-x}} \text{ da für } x \neq -1 \text{ gilt } y = \sqrt{1-x}$$

$$14. \quad y = \left(\frac{x+1}{x-1}\right)^2 \quad \forall x \neq 1 \quad y' = \frac{-4x-4}{(x-1)^3}$$

$$15. \quad \dot{x} = 2 - \frac{1}{t^2} \quad \ddot{x} = \frac{2}{t^3}$$

$$16. \quad y' = \frac{1}{\sqrt{1-x}} + \frac{1+x}{2\sqrt{(1-x)^3}}$$

$$17. \quad y' = \frac{2}{\sqrt{1-x^2}(\sqrt{1+x} + \sqrt{1-x})^2}$$

$$18. \quad y' = \frac{-1}{2\sqrt{a^2-x^2}} - \frac{\sqrt{a-x}}{2\sqrt{(a+x)^3}}$$

$$19. \quad y = (a - \sqrt{2x})^3 \quad y' = -\frac{3(a - \sqrt{2x})^2}{\sqrt{2x}}$$

$$20. \quad y = \frac{1}{2x+3} \quad y' = -\frac{2}{(2x+3)^2}$$

## 4.2 Trigonometrische Funktionen

1.  $y' = x \sin x + 2x + 4$
2.  $y' = -2x \sin(1 - x^2)$
3.  $y' = \frac{\sin 2x}{2 \cos^2 x} + \tan x \cos 2x$
4.  $y' = 6 \sin^2(2x - 3) \cos(2x - 3)$
5.  $y' = (x^2 - 2x + 2) \cos x - 2 \sin x$
6.  $y = x \cos(1 + x) \quad y' = \cos(1 + x) - x \sin(1 + x)$
7.  $y' = \frac{-\cot(x^2)}{x^2} - \frac{2}{\sin^2(x^2)}$
8.  $y' = \frac{\cos x}{x} + \frac{1}{\sin x} - x \frac{\cos x}{\sin^2 x} - \frac{\sin x}{x^2}$
9.  $y' = \frac{(1 + x) \sin x + (1 - x) \cos x}{(\sin x + \cos x)^2}$
10.  $y' = \sin^2 x \cos x (1 - 2 \sin^2 x + \sin^4 x)$
11.  $y' = -9 \sqrt{\frac{1}{3x^3}} \sin^2 \sqrt{\frac{3}{x}} \cos \sqrt{\frac{3}{x}}$
12.  $y' = \frac{6 \cos x}{\sin^2 x} - \frac{9 \cos^3 x}{\sin^4 x} = \frac{-3(\cos^3 x + 2 \cos x)}{\sin^4 x}$
13.  $y' = \frac{7 \sin^3 x}{\cos^8 x}$
14.  $y' = \frac{2 \tan^3(\sqrt{x})}{\sqrt{x} \cos^2 \sqrt{x}}$
15.  $y = \frac{1}{4 \sin x} + \frac{1}{4 \cos x} \quad y' = \frac{1}{4} \left( \frac{\sin x}{\cos^2 x} - \frac{\cos x}{\sin^2 x} \right)$
16.  $y' = -5 \sin \frac{x}{4} \cos^4 \frac{x}{4}$

$$17. \quad y' = \frac{\cos^2 x (1 + \tan x)(\sin x + x \cos x) - x \sin x}{\cos^2 x (1 + \tan x)^2}$$

$$18. \quad y' = \sqrt{\frac{1}{x^3}} \cos \sqrt{\frac{1}{x}} \sin \sqrt{\frac{1}{x}}$$

$$19. \quad y' = \frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} = \tan^2 x + \cot^2 x$$

$$20. \quad y' = 0$$

### 4.3 Logarithmische Ausdrücke, Exponentialfunktionen

$$1. \quad y' = \frac{3x^2}{x^3 + 2} + \frac{2x}{x^2 + 3}$$

$$2. \quad 2yy' = \frac{4}{x} - 2\frac{3}{3x-4} \quad y' = \frac{\frac{2}{x} - \frac{3}{3x-4}}{\pm\sqrt{\ln x^4 - \ln(3x-4)^2}}$$

$$3. \quad y' = \frac{3 \cos 3x}{\sin 3x} = 3 \cot 3x$$

$$4. \quad y' = \frac{1 + \frac{x}{\sqrt{1+x^2}}}{x + \sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$5. \quad y = e^{1-x} \quad y' = -e^{1-x}$$

$$6. \quad y' = 18x$$

$$7. \quad y' = \frac{4a}{(e^{ax} + e^{-ax})^2}$$

$$8. \quad y' = -e^{-x} \ln x + \frac{e^{-x}}{x} \quad y'' = e^{-x} \ln x - 2e^{-x}(1/x + 1/x^2)$$

$$9. \quad y = e^{3x^2 \ln a} \quad y' = a^{3x^2} 6x \ln a$$

10.  $y' = e^{-2x}(3 \cos 3x - 2 \sin 3x)$
11.  $y' = e^{-x^2 \sin x} (-2x \sin x - x^2 \cos x)$
12.  $y' = e^{5x^2-3x+1} (10x - 3)$
13.  $y = e^{x \ln x} \quad y' = x^x (\ln x + 1)$
14.  $y = e^{(\ln x)^2} \quad y' = x^{\ln x} \frac{2 \ln x}{x}$
15.  $y = e^{\ln x \exp(-x^2)} \quad y' = x^{\exp(-x^2)} \exp(-x^2) \left( \frac{1}{x} - 2x \ln x \right)$
16.  $y = 3x^2 \quad y'' = 6x$
17.  $y' = e^{2x} (2x \sin x + 2x^2 e^{2x} + x^2 \cos x)$
18.  $y' = \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$
19.  $y' = x^{(x^x)} (x^x \ln x (1 + x) + x^{x-1})$
20.  $y' = (x^x)^x (2x \ln x + x)$

#### 4.4 Kombinationen verschiedener Funktionen

$$1. \quad y' = x e^{-x^2} (2 \sin x - 2x^2 \sin x + x \cos x)$$

$$2. \quad y' = e^{-(x^2+1) \sin x} (-2x \sin x - (x^2 + 1) \cos x)$$

$$3. \quad y' = \frac{-2x^2 \cos x - \cos x + x \sin x}{e^{-x^2} x^2 \cos^2 x}$$

$$4. \quad y' = (2x^3 \sin^2 x - \cos x \sin x) e^{-x^4}$$

$$5. \quad y' = \frac{1 - 6x^2 \ln x}{x e^{3x^2}}$$

$$6. \quad y' = \cos x e^{-4x^6} (\cos^2 x - 2 \sin^2 x - 24x^5 \sin x \cos x)$$

$$7. \quad y' = -(\cos x)^{\cot x} \left(1 + \frac{\ln \cos x}{\sin^2 x}\right)$$

$$8. \quad y' = \frac{2ab}{a^a \cos^2 x - b^2 \sin^2 x}$$

## BLOCK V

### Integration

1. Berechnen Sie folgende Integrale:

$$(1) \quad \frac{-112}{5}\sqrt{3}$$

$$(2) \quad 0 \text{ (ungerade Funktion)}$$

$$(3) \quad \left[ x - \frac{x^4}{4} \right]_0^2 = 2 - 4 + 0 - 0 = -2$$

$$(4) \quad \ln x + C$$

$$(5) \quad -\cos x + 1$$

$$(6) \quad \left[ \frac{-1}{x} + \frac{1}{2x^2} \right]_{-3}^{-1} = \frac{10}{9}$$

$$(7) \quad u = \sin x \rightsquigarrow \int \frac{1}{u} du = \ln |\sin x| + C$$

$$(8) \quad \text{siehe 7}$$

$$(9) \quad u = 3x^2 - 1 \rightsquigarrow \int \frac{1}{6u} du = \frac{1}{6} \ln |3x^2 - 1| + C$$

$$(10) \quad \ln |x + 1| + C$$

$$(11) \quad \frac{1}{2} \sin(2x + 1) + C$$

$$(12) \quad \int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$(13) \quad \int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

$$(14) \quad -\frac{1}{3}x \cos(3x - 2) + \frac{1}{9} \sin(3x - 2) + C$$

$$(15) \quad \left[ x^2 e^x \right]_0^2 - 2 \int_0^2 x e^x dx = 4e^2 - 2(2e^2 - e^2) - 2 = 2(e^2 - 1)$$

$$(16) \quad \left[ \frac{x^2}{2} \ln x \right]_1^2 - \frac{1}{2} \int_1^2 x dx = 2 \ln 2 - \frac{3}{4}$$

$$(17) \quad e^x + C$$

$$(18) \quad \frac{1}{2} e^x (\cos x + \sin x) + C$$

$$(19) \quad -x^2 \cos x + 2x \sin x + 2 \cos x + C$$

$$(20) \quad x^2 \sin x + 2x \cos x - 2 \sin x + C$$