conventional and the reasons for them will appear in the following discussion:

\[
d_{z^2} = \left( \frac{5}{16\pi} \right)^{1/2} (3 \cos^2 \theta - 1)
\]

\[
d_{xz} = \left( \frac{15}{4\pi} \right)^{1/2} \sin \theta \cos \theta \cos \phi
\]

\[
d_{yz} = \left( \frac{15}{4\pi} \right)^{1/2} \sin \theta \cos \theta \sin \phi
\]

\[
d_{d^2_{z^2}} = \left( \frac{15}{16\pi} \right)^{1/2} \sin^2 \theta \cos 2\phi
\]

\[
d_{xy} = \left( \frac{15}{16\pi} \right)^{1/2} \sin^2 \theta \sin 2\phi
\]

(6.33)

The polar plots of these functions are shown in Figure 6.13.

These five functions are each normalized and mutually orthogonal. The normalization and orthogonality integrals are performed by multiplying a product of functions by \(\sin \theta\) and integrating over \(\theta\) from 0 to \(\pi\) and over \(\phi\) from 0 to \(2\pi\).

The first function is cylindrically symmetric about the \(z\) axis (no \(\phi\) dependence). It vanishes on two cones around the \(z\) axis. These cones are defined by

\[3 \cos^2 \theta = 1\]

which means

\[\cos \theta = \pm 0.57735\]

or

\[\theta = \{ 0.955316 \text{ rad} = 54.7356^\circ \]

\[2.186276 \text{ rad} = 125.2644^\circ \]

If the angular factor is multiplied by \(r^2\), which always appears in the radial wave functions for \(l = 2\), it becomes \(3z^2 - 1\). This accounts for calling this function the \(d_{z^2}\) function. Note that the polar plot consists of two lobes pointed in the +\(z\) and -\(z\) directions and a small "doughnut" in the \(x-y\) plane.

The next two functions of (6.33) form a set, differing only in their \(\phi\) dependence. Multiplying the angular parts by \(r^2\) and using the definitions of polar coordinates, we find \(xz\) from the second function and \(yz\) from the third. Hence their names. The \(d_{yz}\) function is identical to the \(d_{xz}\) function except rotated through \(90^\circ\) about the \(z\) axis. The \(d_{xy}\) function vanishes along the \(y\) axis and has four lobes centered along directions in the \(xz\) plane. These directions bisect the angles formed by the \(x\) and \(z\) axes. The \(d_{d^2_{z^2}}\) function vanishes along the \(x\) axis and has its four lobes centered along directions in the \(yz\) plane. These directions bisect the angles formed by the \(y\) and \(z\) axes.