Bohmian Mechanics with Complex Action:
A New Trajectory Based Formulation of Quantum Mechanics

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Ever since the advent of Quantum Mechanics, there has been a quest for a trajectory based formulation of quantum theory that is exact. In the 1950's, David Bohm, building on earlier work of Madelung and de Broglie, developed an exact formulation of quantum mechanics in which trajectories evolve in the presence of the usual Newtonian force plus an additional quantum force. In recent years, there has been a resurgence of interest in Bohmian Mechanics (BM) as a numerical tool because of its apparently local dynamics, which could lead to significant computational advantages for the simulation of large quantum systems. However, closer inspection of the Bohmian formulation reveals that the nonlocality of quantum mechanics has not disappeared --- it has simply been swept under the rug into the quantum force. In this work, we present a new formulation of Bohmian mechanics in which the quantum action, *S*, is taken to be complex. This requires the propagation of complex trajectories, but with the reward of a significantly higher degree of localization. For example, using strictly localized trajectories (no communication with their neighbors) we are to obtain virtually exact quantum mechanical tunneling probabilities down to 10^{-7}. We have recently extended the formulation to allow us to obtain interference effects, which has been one of the major obstacles in conventional Bohmian mechanics. We show that by allowing crossing of local, complex trajectories, the new formulation leads to wavefunction nodal patterns as a sum of the contribution from several crossing trajectories. We demonstate the formulation with applications to one- and two-dimensional tunneling, thermal rate constants in one and two dimensions, and the calculation of eigenvalues. On the formal side, the approach is shown to be a rigorous extension of semiclassical Gaussian wavepacket methods to give exact quantum mechanics, and has intriguing implications for fundamental quantum mechanics.

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