# Optimal Design of Private and Occupational Retirement Plans

## Summary

Cumulative Doctoral Thesis at Ulm University
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This cumulative thesis contributes to the field of optimal retirement planning, optimal retirement product design and optimal asset allocation in the context of collective investment problems. It consists of the following five research papers:

- Chen, A. and Rach, M. (2019). Options on tontines: An innovative way of combining annuities and tontines. *Insurance: Mathematics and Economics*, 89:182–192. Available at: `https://doi.org/10.1016/j.insmatheco.2019.10.004`
- Chen, A., Rach, M., and Sehner, T. (2020). On the optimal combination of annuities and tontines. *ASTIN Bulletin: The Journal of the IAA*, 50(1):95–129. Available at: `https://doi.org/10.1017/asb.2019.37`
- Chen, A., Hieber P., and Rach, M. (2019). Optimal retirement products under subjective mortality beliefs. *Insurance: Mathematics and Economics* (The thesis contains an earlier version of the now published article). Published article available at: `https://doi.org/10.1016/j.insmatheco.2020.07.002`
- Chen, A., Nguyen, T., and Rach, M. (2019). Optimal collective investment: The impact of sharing rules, management fees and guarantees. *Journal of Banking and Finance*, 123, 106012 (The thesis contains an earlier version of the now published article). Published article available at: `https://doi.org/10.1016/j.jbankfin.2020.106012`
- Chen, A., Nguyen, T., and Rach, M. (2019). A collective investment problem in a stochastic volatility environment: The impact of sharing rules. *Annals of Operations Research* (The thesis contains an earlier version of the now published article). Published article available at: `https://doi.org/10.1007/s10479-021-03983-8`

## Motivation

In Germany and many other developed countries, the retirement system can be divided into three pillars: mandatory state pension systems, occupational pensions and private pensions. An aging society and prevailing low interest rates lead to a reduced retirement income from the first pillar, forcing individuals to plan appropriately for retirement in the second and third pillar. As an example, Figure 1 illustrates the composition of the German population in 2018, showing that roughly 50% of the citizens are currently aged 45 or older. To deal with these

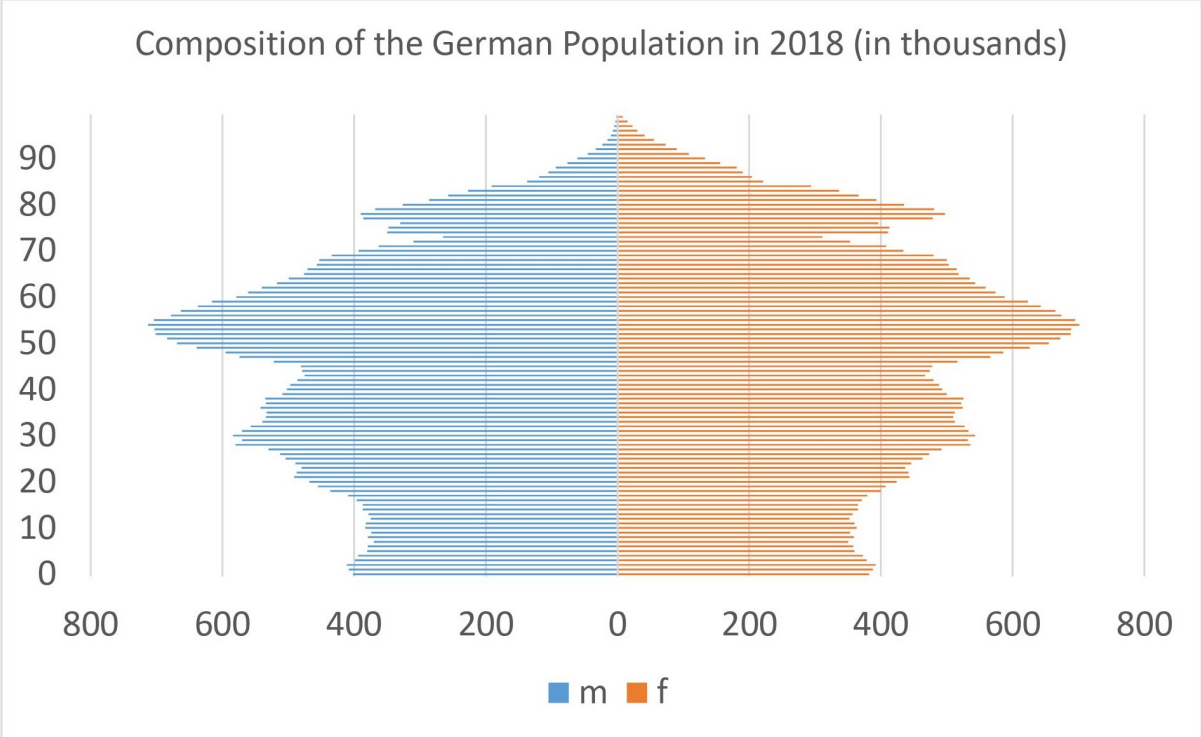


Figure 1: German population 2018 separated by age and gender (m = male, f = female). Data source: Statistisches Bundesamt (Destatis) (2019).

societal challenges, many countries go through some pension reforms and new retirement plans are developed both in the second and third pillar.

One of the recent developments has been the resurrection of the so-called *tontine*, named after its inventor Lorenzo de Tonti, which used to be a popular source of retirement income from the 17th to the 19th century. The idea of a tontine is that a group of policyholders shares the mortality risk and that the insurance company only serves as an administrator. Another development in the light of the current societal challenges is that, in most developed countries, there has been a shift from defined benefit (DB) towards defined contribution (DC) plans, i.e. the market risk has been shifted fully from the employers to the employees. Naturally, the

questions will be asked whether such a drastic risk transfer might worsen the benefits of the employees too substantially, and whether hybrid plans combining the advantages of DC and DB plans are the future of occupational pension schemes.

## Contributions and selected results

The first three research papers in this thesis focus on the third pillar, analyzing the potential of tontines and related products. Starting with Yaari (1965), expected utility has been frequently used in the literature to find optimal payoff structures or to compare different retirement plans from a policyholder's perspective. When priced actuarially fair, life annuities give retirees greater lifetime utility than tontines, a result which can be reversed if safety loadings are charged (cf. Milevsky and Salisbury (2015)). Table 1 provides a brief characterization of annuities and tontines from the policyholder's and insurer's perspective.

| | Annuity | Tontine |
|---|---|---|
| Policyholder | • Stable, guaranteed payments<br>• High prices | • Volatile payments<br>• Cheaper than an annuity |
| Insurer | • High risk capital requirement<br>• Low demand ("Annuity puzzle") | • Lower risk capital requirement<br>• Higher demand? |

Table 1: Characterization of annuities and tontines.

In the first two articles, we consider possible combinations of annuities and tontines which aim at combining the advantages of the original products. In the first paper, we construct a tontine with minimum guaranteed payments and show that such a product can generate a higher expected lifetime utility than an annuity and a tontine for policyholders. Additionally, we show that insurers selling the new retirement product face a lower conditional expected loss but an (almost) identical loss probability compared to insurers selling annuities. In the second paper, we compare the innovative retirement plan tonuity introduced in Chen et al. (2019) to two additional combinations of annuities and tontines: One which we call antine, working like a reversed tonuity, and a portfolio consisting of a tontine and an annuity. We find that, under reasonable safety loadings, a simultaneous investment in a tontine and an annuity delivers a higher expected lifetime utility than the single products. The tonuity and antine can be replicated by a portfolio of a tontine and an annuity and, thus, deliver a lower expected lifetime utility than the portfolio.

The third article is motivated by Smith (1776) who pointed out that an over- or underestimation

of one's remaining lifetime affects the perceived attractiveness of retirement products. In the third article, we find that, even under actuarially fair premiums, tontines can be preferred over annuities under subjective mortality beliefs. The main driving factor behind the tontine's relative attractiveness is that the policyholder underestimates the remaining lifetimes of other policyholders relative to the remaining lifetimes assumed by the insurer.

The main application of the last two papers is the second pillar. Using an expected utility framework, it is possible to derive the optimal continuous-time trading strategy for an employee in the accumulation phase. For a single investor, this procedure is well-documented and dates back to Merton (1969, 1971). Extending Merton's pioneering articles, different types of portfolio insurance constraints have been considered, typically taking the perspective of a single investor. However, frequently individual plan members contribute to a collectively organized pension fund instead of handling the investments of their contributions on their own.

In the fourth paper, we therefore consider a collective of individuals with heterogeneous risk preferences who are tied together in their investment decisions and delegate a fund manager to invest their wealth in a Black-Scholes financial market under portfolio insurance. Concerning the portfolio insurance constraint, we consider two cases: a deterministic and a flexible, state-dependent guarantee. We include management fees and assume that the investors in the collective can be divided into two groups, each with a different willingness to pay the fee. We show that deterministic guarantees embedded in DC schemes deteriorate the benefits of employees and that a more flexible guarantee can increase the expected utility compared to the frequently prescribed deterministic guarantee. To make the collective investment even fairer, careful considerations shall be given to the sharing rules and the communication about management fees. Finally, in the fifth paper, we set ourselves in a more realistic financial market with stochastic volatility. Additionally, we directly include the demand for guarantees in the utility functions of the investors. We find that individual optima are achievable in the collective, given that a financial fairness criterion and a state-dependent sharing rule are used. Under more practical linear sharing rules, a market with stochastic volatility leads to higher losses for each individual compared to the Black-Scholes model.

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