A Coupled Volume-of-Fluid/ Level Set Method in OpenFOAM

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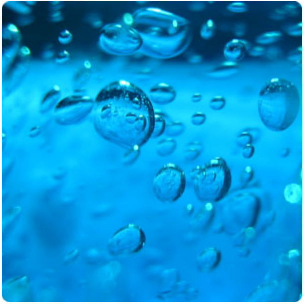
Overview

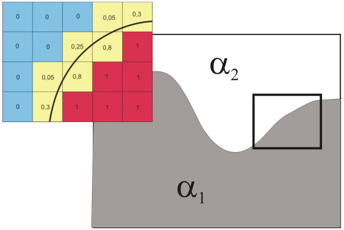
Motivation

- Representation of sharp interfaces to capture phenomena at the interface
- Application of a Surface Capturing method
- OpenFOAM framework



Volume-of-Fluid in OpenFOAM®

- volumetric phase-fraction α_1:



phase 1: $\alpha_1 = 1$
phase 2: $\alpha_1 = 0$
interface: $0 < \alpha_1 < 1$

- Capturing of the interface

$$\frac{\partial \alpha_1}{\partial t} + \nabla \cdot (\mathbf{U}\alpha_1) + \nabla \cdot (\mathbf{U}_r \alpha_1 \alpha_2) = 0$$

relative velocity at the interface

$$\mathbf{U}_r = \min\left(c_{\alpha_1} |\mathbf{U}|, \max(|\mathbf{U}|)\right) \cdot \mathbf{n}$$

Compression of the interface controlled by parameter c_α

Volume-of-Fluid in OpenFOAM®

- phase averaged material properties

$$\rho = \sum_{i=1}^{N} (\alpha_i \rho_i), \quad \mu = \sum_{i=1}^{N} (\alpha_i \mu_i)$$

- continuity equation

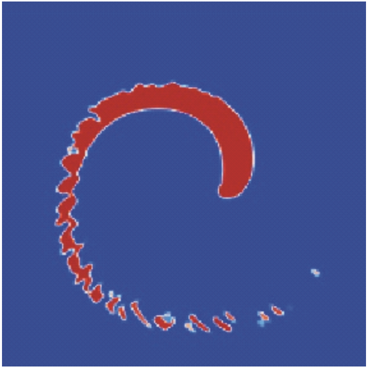
$$\nabla \cdot \mathbf{U} = 0$$

- mixture approach
  ⇒ one equation to describe the transport of momentum

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot (\rho \mathbf{U}\mathbf{U}) = -\nabla p^* + \nabla \cdot (\mu \nabla \mathbf{U}) + (\nabla \mathbf{U} \cdot \nabla \mu) - \mathbf{f} \cdot \mathbf{x} \nabla \rho + \sigma \kappa \nabla \alpha_1$$

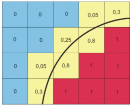
Volume-of-Fluid in OpenFOAM - problems of the implemented approach

- diffusing interface
- unphysical behaviour during elongation



Volume-of-Fluid and Level-Set

Volume of Fluid [7]

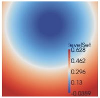


- volumetric phase fraction α:

phase 1	$\alpha = 1$
phase 2	$\alpha = 0$
interface	$0 < \alpha < 1$

- transport of α:
  $\frac{\partial \alpha}{\partial t} + \nabla \cdot (\mathbf{U}\alpha) = 0$

Level Set [4]



- Level-Set function ϕ:
  distance between cell-centre and interface
- transport von ϕ:
  $\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{U}\phi) = 0$

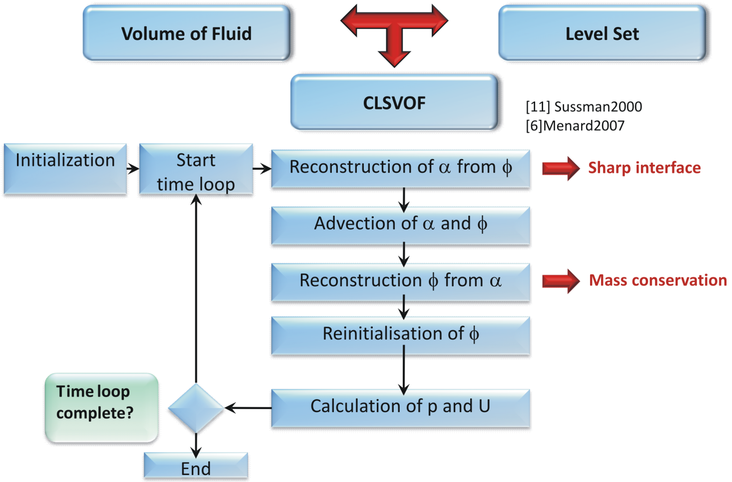
phase averaged momentum balance

$$\frac{\partial(\rho\mathbf{U})}{\partial t} + \nabla \cdot (\rho\mathbf{U}\mathbf{U}) = -\nabla p^* + \nabla \cdot (\mu\nabla\mathbf{U}) + (\nabla\mathbf{U} \cdot \nabla\mu) - \mathbf{f} \cdot \mathbf{x}\nabla\rho + \sigma\kappa\nabla\alpha_1$$

- mass-conservative
- diffusion of the interface

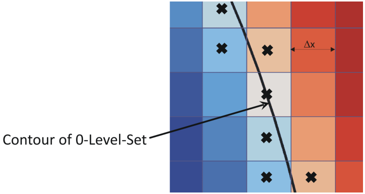
- sharp interface
- not mass-conservative

Coupled-Level-Set/Volume-of-Fluid (CLSVOF) Approach



Reconstruction of the Interface

1. identification of the cells describing the interface
   - cells with $\phi < (0.5\sqrt{\Delta x})$ (depending on the position of the interface) $\quad 0 < \alpha < 1$
   - $sign(\phi) = -1 \quad \alpha = 1$
   - $sign(\phi) = 1 \quad \alpha = 0$



2. linear reconstruction of α from ϕ

$$p := a_{i,j,k}(x - x_i) + b_{i,j,k}(y - y_j) + c_{i,j,k}(z - z_k) + d_{i,j,k}$$

plane p representing the 0-contour of the Level-Set variable

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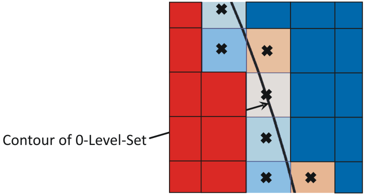

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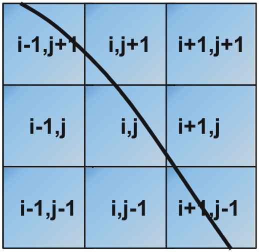
$$p := a_{i,j,k}\,(x - x_i) + b_{i,j,k}\,(y - y_j) + c_{i,j,k}\,(z - z_k) + d_{i,j,k}$$

plane p representing the 0-contour of the Level-Set variable

Reconstruction of the interface

1. computation of the plane coefficients $a_{i,j,k}, b_{i,j,k}, c_{i,j,k}, d_{i,j,k}$ by error minimation

$$E_{i,j,k} = \sum_{i'=i-1}^{i'=i+1} \sum_{j'=j-1}^{j'=j+1} \sum_{k'=k-1}^{k'=k+1} w_{i'-i,j'-j,k'-k} \delta\left(\phi_{i',j',k'}\right) \left(\phi_{i',j',k'} - a_{i,j,k}\left(x_{i'} - x_i\right) - b_{i,j,k}\left(y_{j'} - y_j\right) - c_{i,j,k}\left(z_{k'} - z_k\right) - d_{i,j,k}\right)$$



2. weighting functions
   central cell: $w = 16$
   sourrounding cells: $w = 1$

Reconstruction of the interface

3. computation of the coefficients $a_{i,j,k}, b_{i,j,k}, c_{i,j,k}, d_{i,j,k}$ by solving the corresponding linear equation system

$$\begin{bmatrix} \sum\sum\sum whX^2 & \sum\sum\sum whXY & \sum\sum\sum whXZ & \sum\sum\sum whX \\ \sum\sum\sum whXY & \sum\sum\sum whY^2 & \sum\sum\sum whYZ & \sum\sum\sum whY \\ \sum\sum\sum whXZ & \sum\sum\sum whYZ & \sum\sum\sum whZ^2 & \sum\sum\sum whZ \\ \sum\sum\sum whX & \sum\sum\sum whY & \sum\sum\sum whZ & \sum\sum\sum wh \end{bmatrix} \cdot \begin{bmatrix} a_{i,j,k} \\ b_{i,j,k} \\ c_{i,j,k} \\ d_{i,j,k} \end{bmatrix} = \begin{bmatrix} \sum\sum\sum whX\phi \\ \sum\sum\sum whY\phi \\ \sum\sum\sum whZ\phi \\ \sum\sum\sum wh\phi \end{bmatrix}$$

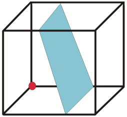
with the abbreviations

$$\begin{aligned} wh &= w_{i-i',j-j',k-k'} H\left(\phi_{i',j',k'}\right) \\ X &= x_{i'} - x_i \\ Y &= y_{i'} - y_i \\ Z &= z_{i'} - z_i \end{aligned}$$

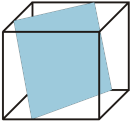
Reconstruction of the interface

Computation of the phase fraction

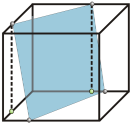
1. transformation of the coordinate system



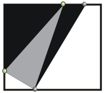
2. representation of the plane as $Z = AX + BY + C$



3. calculation of the cutting points between plane and cubus
4. projection of the cutting points into plane z=0



5. definition of triplets
6. integration over the triplets



$$V = \int_{x_i}^{x_j} \int_{y(x_k)}^{y(x_k)} AX + BY + Cdydx$$

Reconstruction of ϕ from α

Solving the inverse problem

- definition of the volumetric phase fraction as function of the Level-Set variable

$$\frac{1}{dxdydz}\int_{z_{k-1/2}}^{z_{k+1/2}}\int_{y_{j-1/2}}^{y_{j+1/2}}\int_{x_{i-1/2}}^{x_{i+1/2}} H\left(a_{i,j,k}\left(x-x_i\right)+b_{i,j,k}\left(y-y_j\right)+c_{i,j,k}\left(z-z_k\right)+d_{i,j,k}\right) dxdydz = \alpha_{i,j,k}$$

- if mass-conservation is not fulfilled after advection of ϕ und α: adaption of the parameter $d_{i,j,k}$

$$d_{i,j,k}^{New} = d_{i,j,k} - \frac{\frac{1}{dxdydz}\int_{z_{k-1/2}}^{z_{k+1/2}}\int_{y_{j-1/2}}^{y_{j+1/2}}\int_{x_{i-1/2}}^{x_{i+1/2}} H\left(a_{i,j,k}\left(x-x_i\right)+b_{i,j,k}\left(y-y_j\right)+c_{i,j,k}\left(z-z_k\right)+d_{i,j,k}\right) dxdydz}{\int_{z_{k-1/2}}^{z_{k+1/2}}\int_{y_{j-1/2}}^{y_{j+1/2}} + \int_{x_{i-1/2}}^{x_{i+1/2}} H\left(a_{i,j,k}\left(x-x_i\right)+b_{i,j,k}\left(y-y_j\right)+c_{i,j,k}\left(z-z_k\right)+d_{i,j,k}\right) dxdy}$$

- re-evaluation of ϕ

CLSVOF-Class

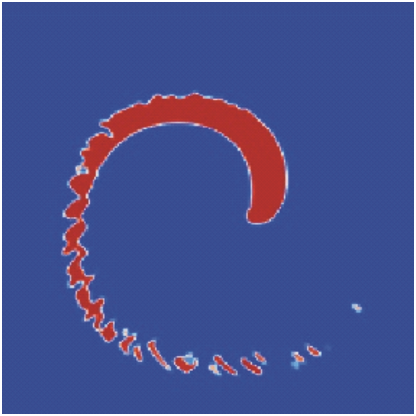
- Heaviside-function
- calculation of the plane-coefficients

- transformation of the coordinate-system
- geometrical calculation of the cutting-points
- calculation of the phase-fractions and the distribution of the Level-Set function

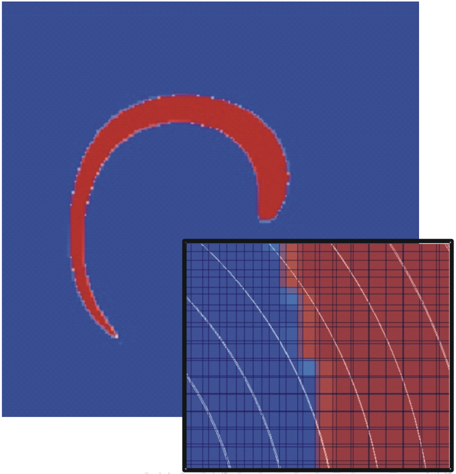
- solution of the transport equations for ϕ and α
- reconstruction of the Level-Set field as a signed distance function

Circle in a Vortex

normal implementation

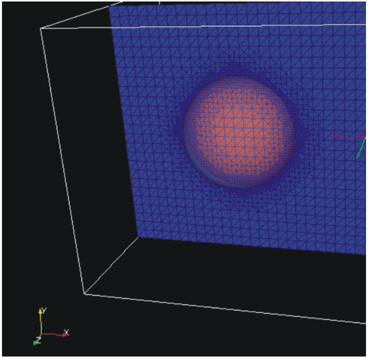


interCLSVOFFoam



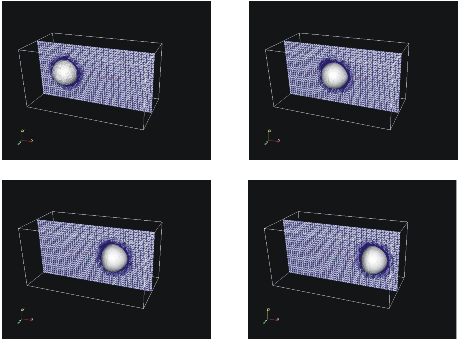
Results

- adaptive mesh refinement to achieve high resolution
- droplet under influence of gravity



Results

- transport of a droplet in a channel under zero gravity conditions



Summary and Outlook

Summary

- implementation of the CLSVOF-method in OpenFOAM
- sharp interface

Outlook

- still a small mass-conservation problem
  $\longrightarrow$ implementation of a 5th order WENO scheme
- parallelization