Reynolds transport theorem:

\[
\frac{d}{dt} \int_{\Omega(t)} f(x,t) \, d\Omega = \int_{\Omega(t)} \left\{ \frac{\partial f}{\partial t}(x,t) + \nabla \cdot (f \, \vec{u}) \right\} \, d\Omega
\]

**Preliminaries** Let \( \Omega_0 \subset \mathbb{R}^n \) and let \( \phi : \Omega_0 \times [0, \infty) \rightarrow \Omega(t) \subset \mathbb{R}^n \) be the transformation defined by \( (x,t) \mapsto y := \phi(x,t) \).

\[
\Rightarrow \frac{d}{dt}y =: u(y,t) = u(\phi(x,t),t) = \frac{d}{dt}\phi(x,t) =: \phi_t(x,t)
\]

Calculation of the Jacobian-matrix:

\[
D_x \phi_t(x,t) =: \phi_{xt}(x,t) \Rightarrow \phi_{tx}(x,t) = u_x(\phi(x,t),t) \cdot \phi_x(x,t)
\]

**Proof:** For a more detailed proof we refer to the lecture notes of Prof. Dr. J. Lorenz, “Die Naviers-Stokes Gleichung ..”, RWTH-Aachen, 1994.

1. Transformation theorem with respect to \( \phi(t, \cdot) : \Omega_0 \rightarrow \Omega(t) \)

2. Derivation rule for Wronski determinants: (würde hier das ganze Lemma hinschreiben sonst ist nicht klar was \( Y(t) \) bzw. \( A(t) \) überhaupt ist.) \( \frac{d}{dt} \det Y(t) = \text{tr} A(t) \cdot \det Y(t) \)

\[
\frac{d}{dt} \det (\phi_x(x,t)) = \text{tr} u_x(\phi(x,t),t) \det (\phi_x(x,t))
\]

and

\[
\text{tr} u_x(\phi(x,t),t) = (\nabla \cdot u)(\phi(x,t),t)
\]