Computational Biomechanics 2017

Lecture I:

Introduction, Basic Mechanics 1

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Scientific Computing Centre Ulm, UZWR
Ulm University

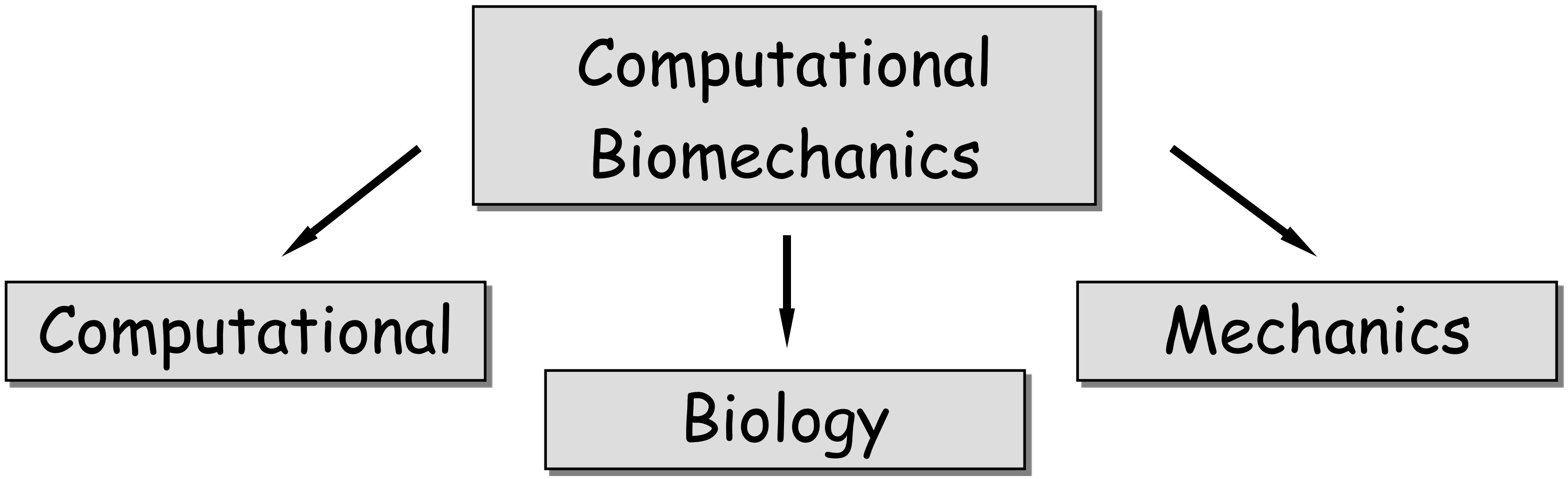
0 Organisation

Scientific Computing Centre Ulm

→ www.uzwr.de

- Rerun
- Lecture times
- Exam
- Max. 12 students

1 General Information



Biomechanics: Solving biological questions using methods of mechanical engineering (Technische Mechanik), incl. experiments.

Mechanobiology: Reaction of biological structures on mechanical signals. Mechanotransduction: Melekular cell reaction.

Research Fields

Orthopaedic Biomechanics: Bone-implant contact, fracture healing, (artificial) joints, musculoskeletal systems, …

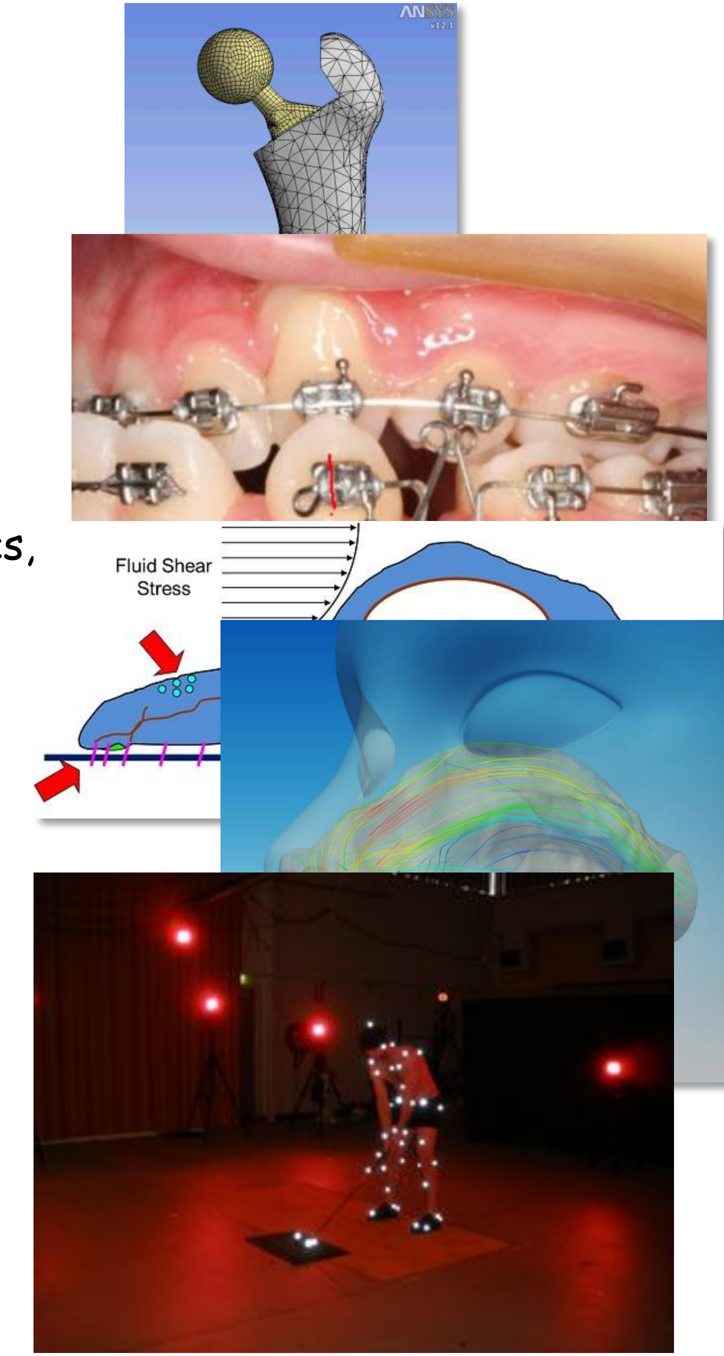
Dental Biomechanics: Dental implants, orthodontics, dental movements, braces, brackets, …

Cell Biomechanics: Cell experiments (cell gym) and simulations to study mechenotransduction

Fluid Biomechanics: Respiratory systems, blood flow, heart, …

Sport Biomechanics: Optimizing performance, techniques and equipment of competitive sports

Tree Biomechanics, Traffic Safety, Accident Research, …



Numerical Methods

Boundary Value Problems: Finite Elements, static structural analyses, displacements, stresses & strains,

Initial Value Problems: Forward dynamics problem, multi-body systems, musculoskeletal systems, movements, inverse dynamics problem: Calculating muscle forces from measured movements

Multiscale Modeling: To handle higly complex systems

Model Reduction: dito

Fuzzy Logic: Fracture healing in Ulm

Contents

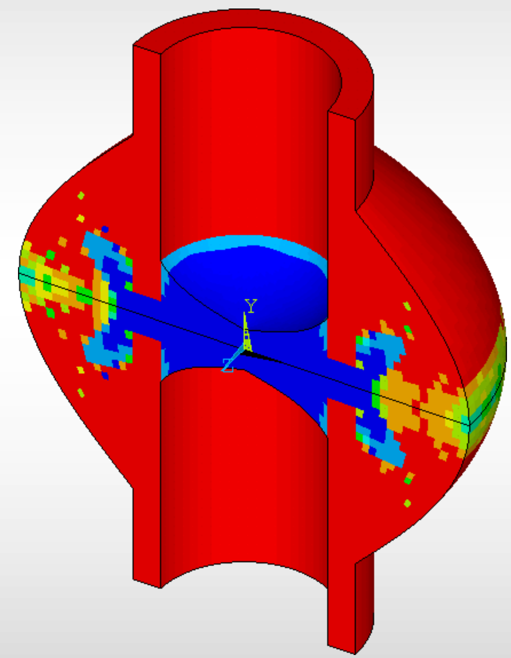
Detailed Schedule Summer 2017

SW	Day	Date	Topic	Lecturer
01	Mo	17 Apr	- Holiday -	-
02	Mo	24 Apr	L01: Intro to Biomechanics; Mech 1: Statics	Ulli
03	Mo	01 May	- Holiday -	-
04	Mo	08 May	L02: Mech 2: Elastostatics; Mat. Props. Biol. Tissues	Ulli
04	Mo	02 May	L03: Intro FEA, Intro to Ansys WB, Lab: Trabec. B. 1	Ulli
05	Mo	09 May	L04: Geometry from Imaging Data (1/2), Lab: Trabec. B. 2	Frank
06	Mo	16 May	- Holiday: Pentecost -	-
07	Mo	23 May	L05: Geometry from Imaging Data (2/2); Exercise: FE from CT data	Frank
08	Mo	30 May	L06: Forward & Inverse Dynamics Part 1; ADAMS Exercise	Ulli
09	Mo	06 Jun	L07: Forward & Inverse Dynamics Part 2; Anybody Demonstration	Ulli
10	Mo	13 Jun	L08: Bone Remodeling (Biology, Theories & Models)	Frank
11	Mo	20 Jun	L09: Fracture Healing 1 (Basics & Simulation)	Frank
12	Mo	27 Jun	L10: Fracture Healing 2: Level-Set Method	Martin
13	Mo	04 Jul	L11: CFD 1: Theory, Numerics, Modelling	Martin
14	Mo	11 Jul	L12: CFD 2: Human Nose Air Flow	Martin
+1	Mo	25 Jul	Oral Examinations, 14:15, Office Simon, UZWR	All

1.2 Appetizer:

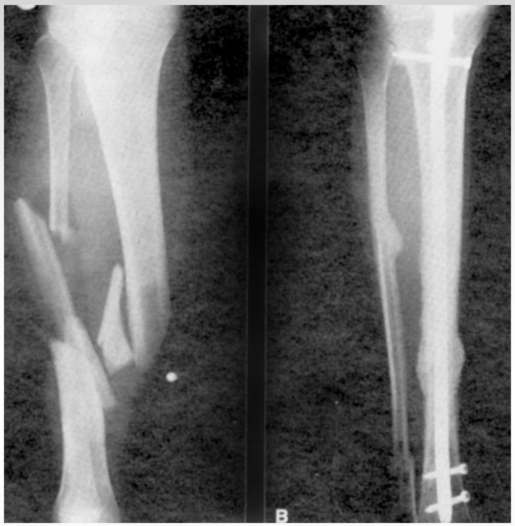
Simulation of Bone Healing

Computermodell für die Knochenheilung



Introduction: Bone Healing

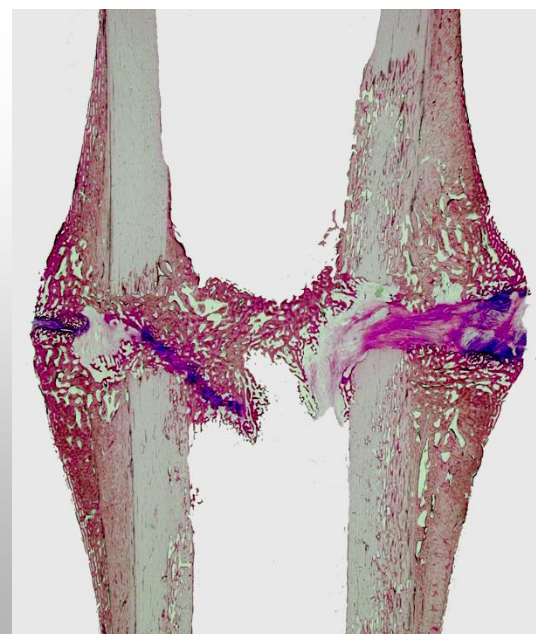
Mechanical situation



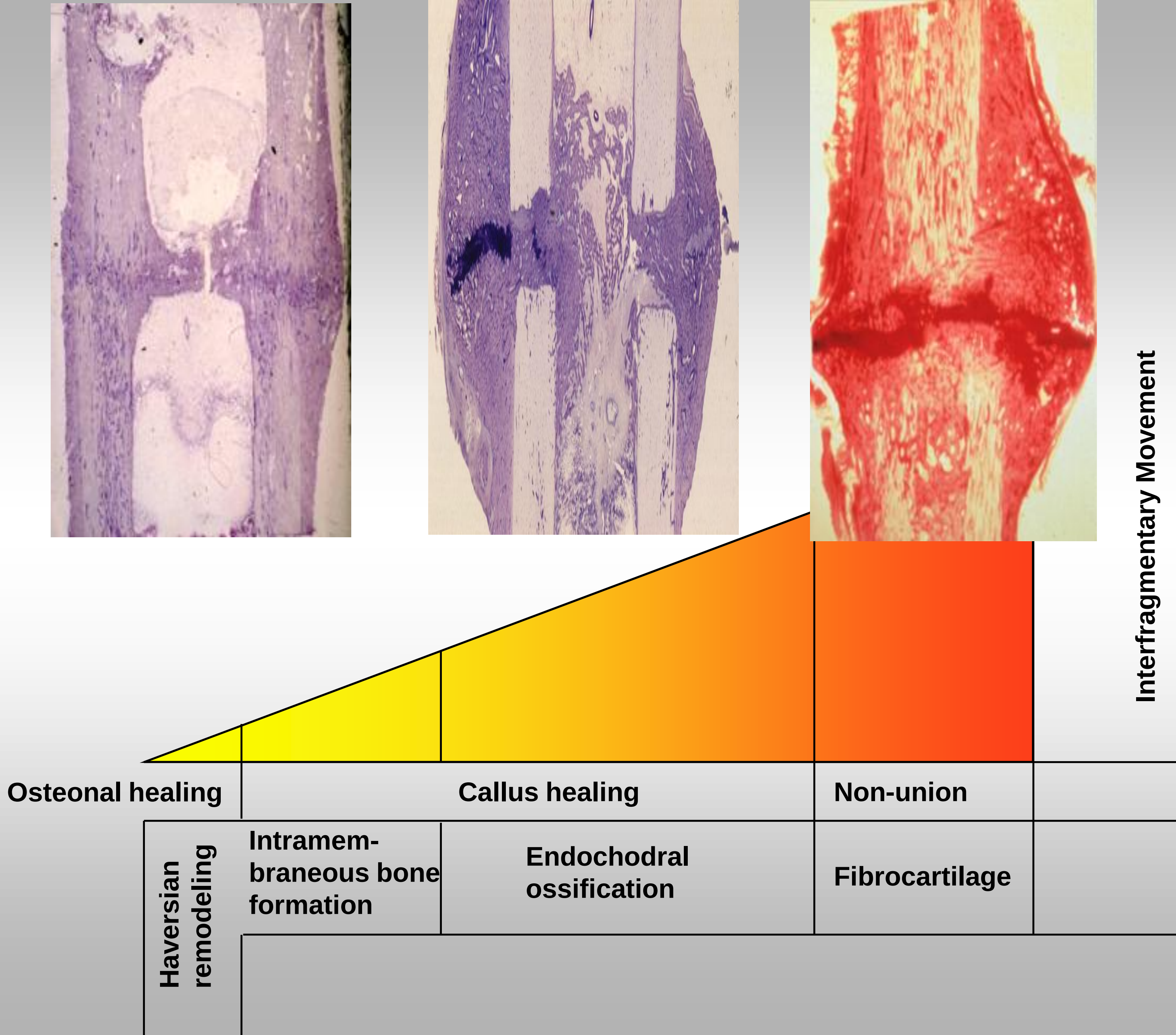
Local Blood supply

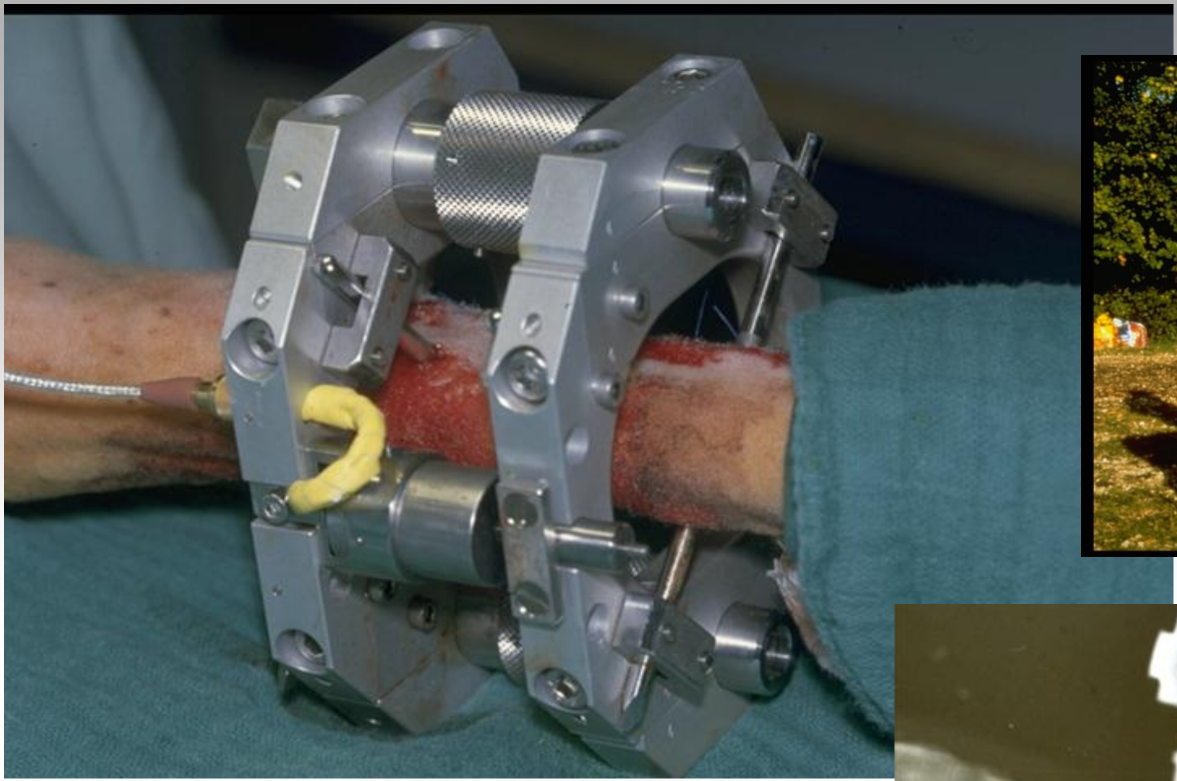
Rhinelander (1968)*

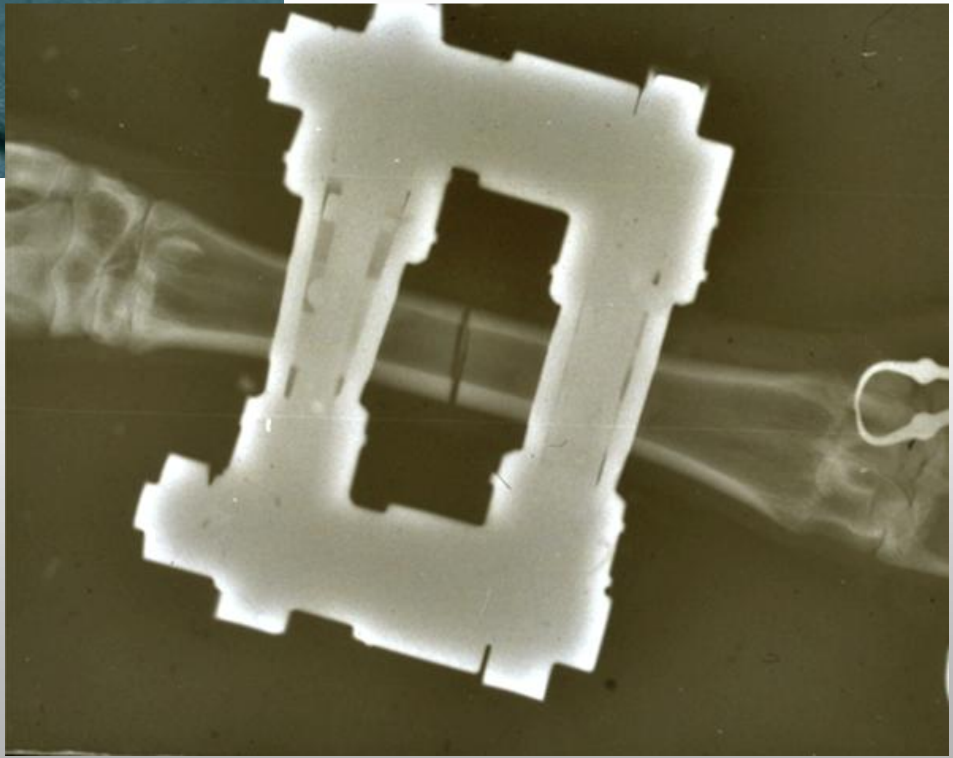
Healing process



*) From Bone in Clinical Orthopedics by permission of AO Publishing.

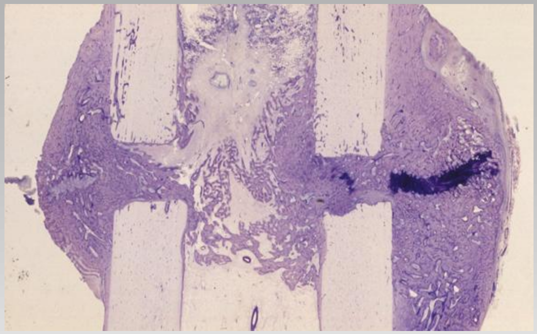

Interfragmentary Movement
Osteonal healing
Callus healing
Non-union
Haversian remodeling
Intramem-braneous bone formation
Endochodral ossification
Fibrocartilage

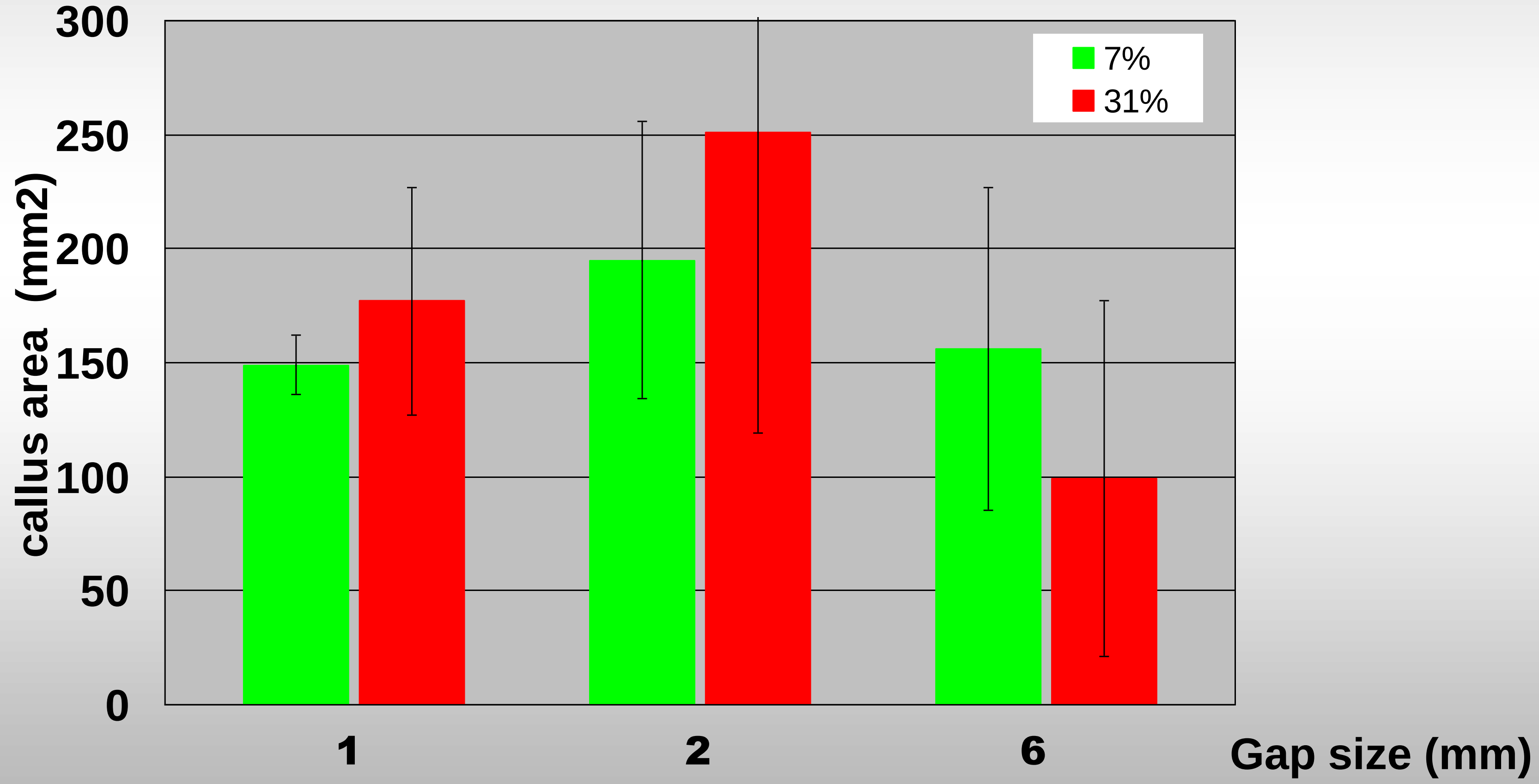




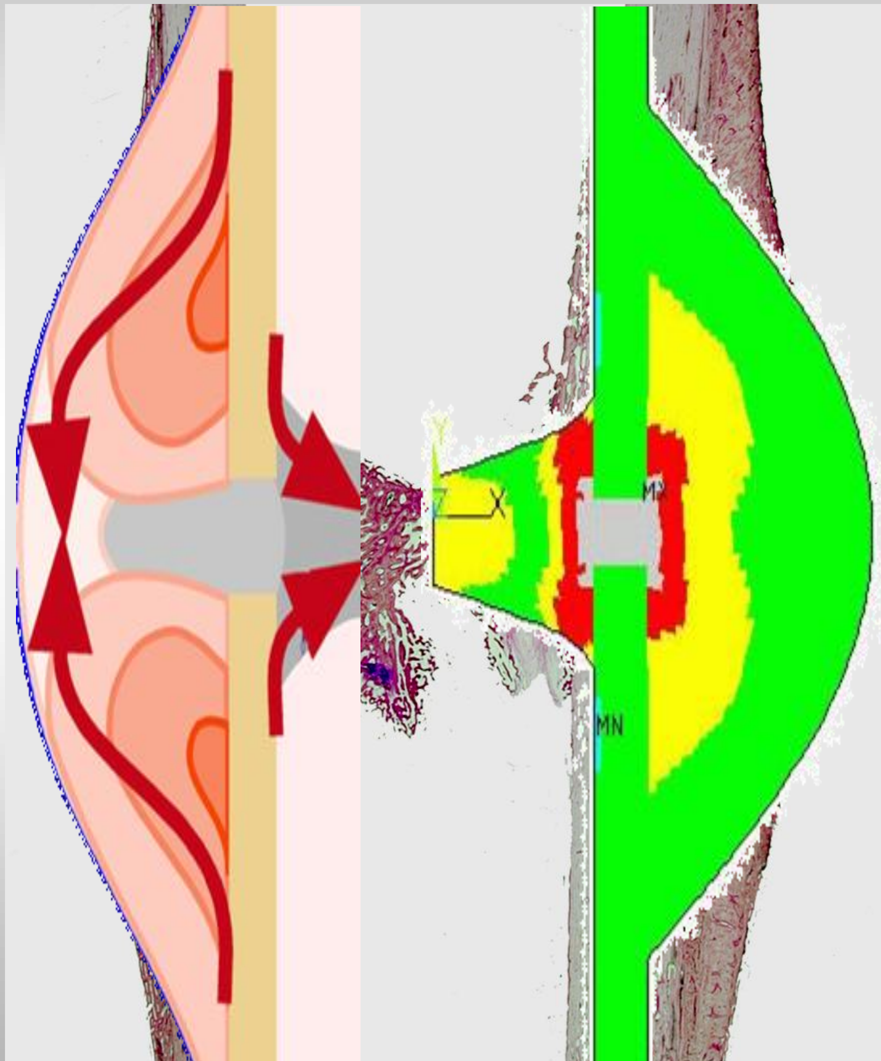
Animal Experiments
- Sheep methatarsus
- Fixateur exteren
- Gap sizes: 1, 2, 6 mm
- Interfrag. Strain: 7%, 31%

Exp. Results: Callus Area





Introduction: Callus Healing Process

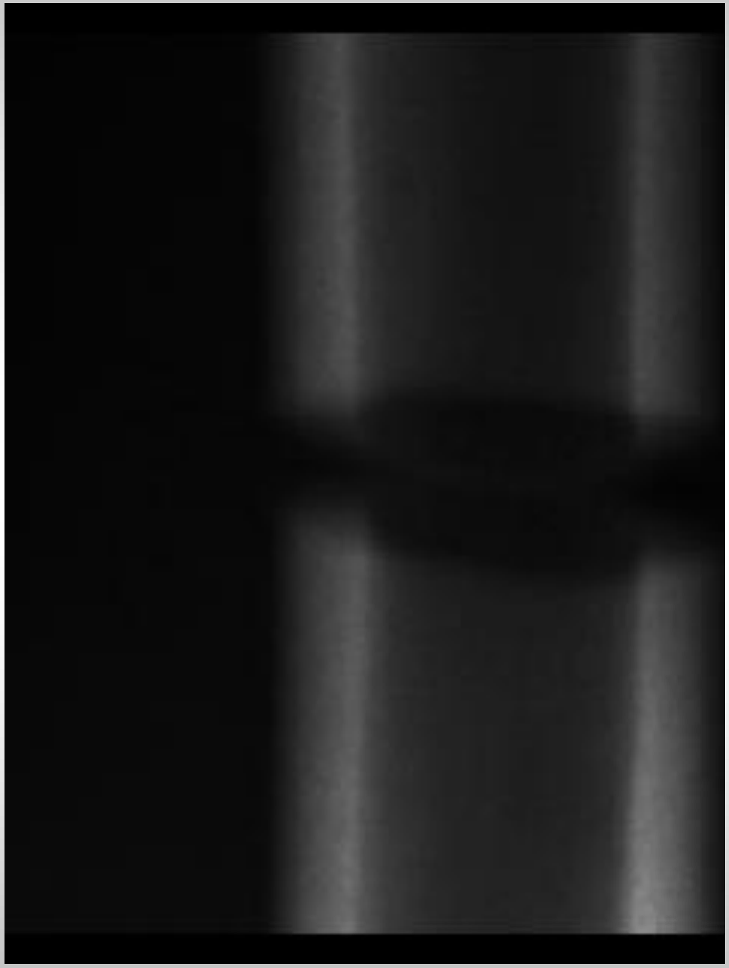


New bone formation only with low strain state !

Biological Solution:

1. Callus: larger diameter
2. Via cartilage to bone

Wirklichkeit

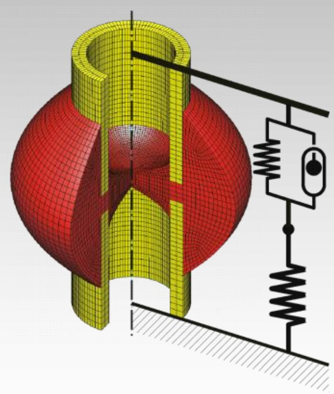


Modell

Methods: Finite Element Model

Geometry:

- Idealized osteotomy (sheep)
- 2D, axi-symmetric
- Predescribed, fixed healing area



Discretization:

- 3.000 el.

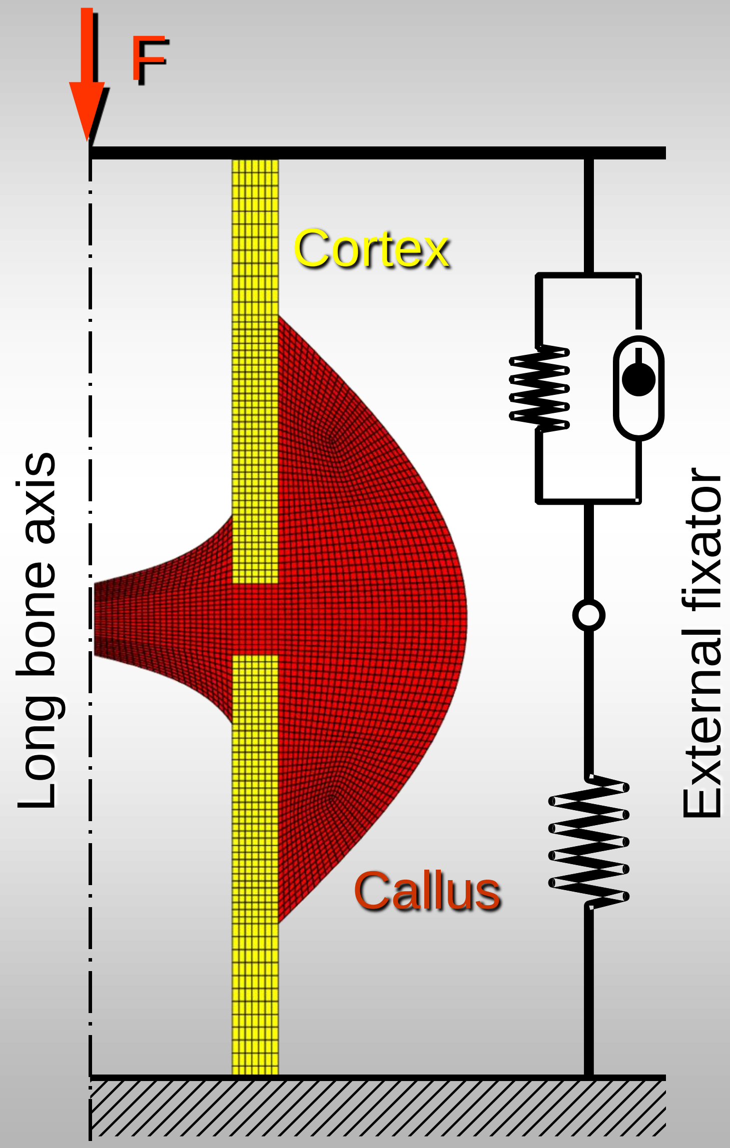
Material properties:

Tissue type	*Modul*	*Poisson ratio*
Cortical bone	10,000 MPa	0.36
Woven bone	4,000 MPa	0.36
Fibro cartilage	200 MPa	0.45
Connective tissue	3 MPa	0.30

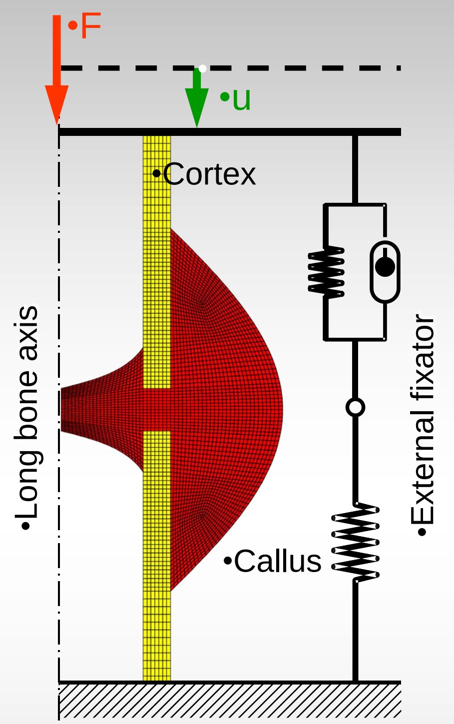
Load / Boundary conditions (Claes 97):

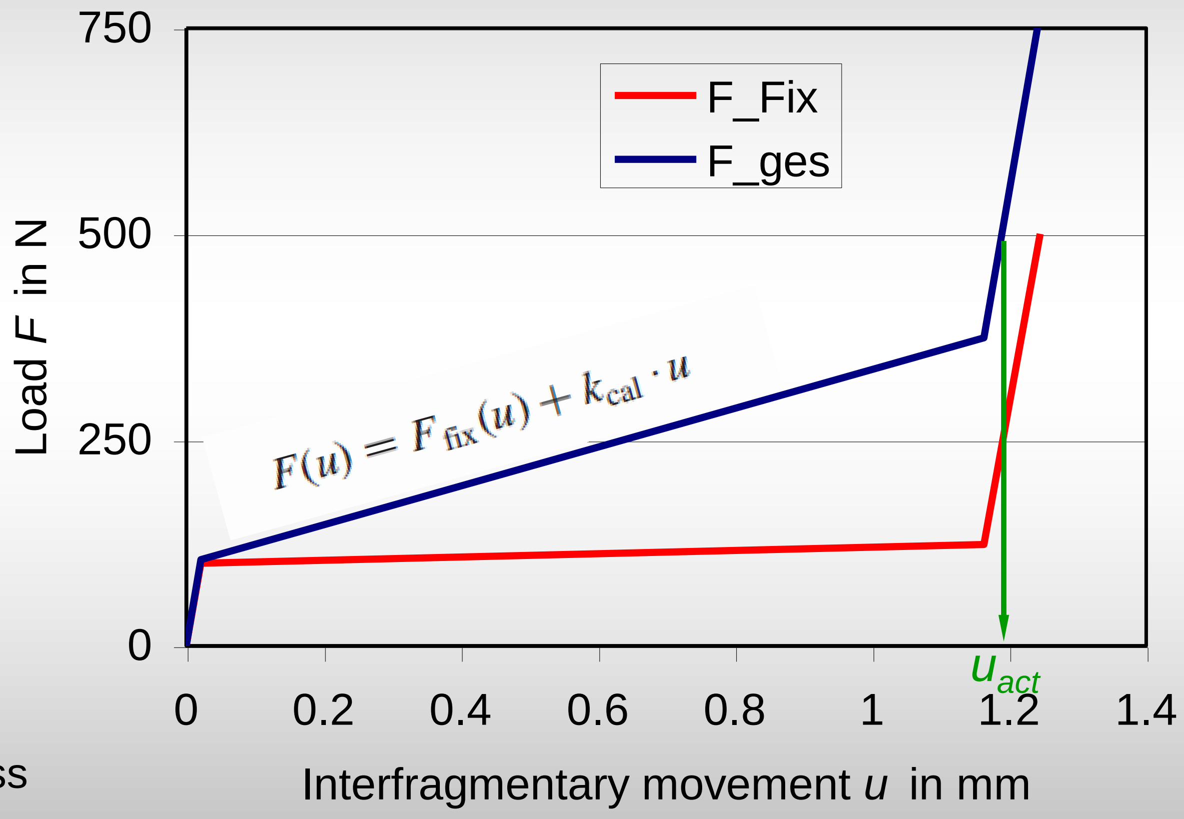
<u>Case A</u>: Initial movement = 0.25 mm

<u>Case B</u>: Initial movement = 1.30 mm



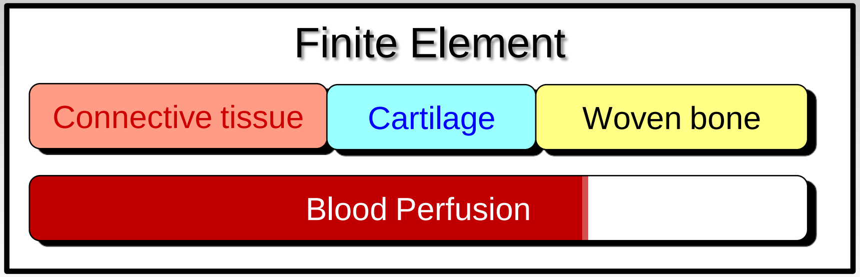
Separation





- FEA of linera part: callus from unit displacement u_{unit}
- Calculation of callus stiffness
- Calculation of F(u)
- Using inverse u(F) to find u_{actual}
- Scale callus strains with u_{actual}/u_{unit}

Methods: Tissue Mixture



Volumetric Tissue Concentrations

Perfusion Index

Constraint

$$\sum_{\text{tiss}} c_{\text{tiss}} \stackrel{!}{=} 1, \quad \text{tiss} = (\text{soft}, \text{cart}, \text{bone}).$$

Resultant State Variables

$0 \leq c_i \leq 1$

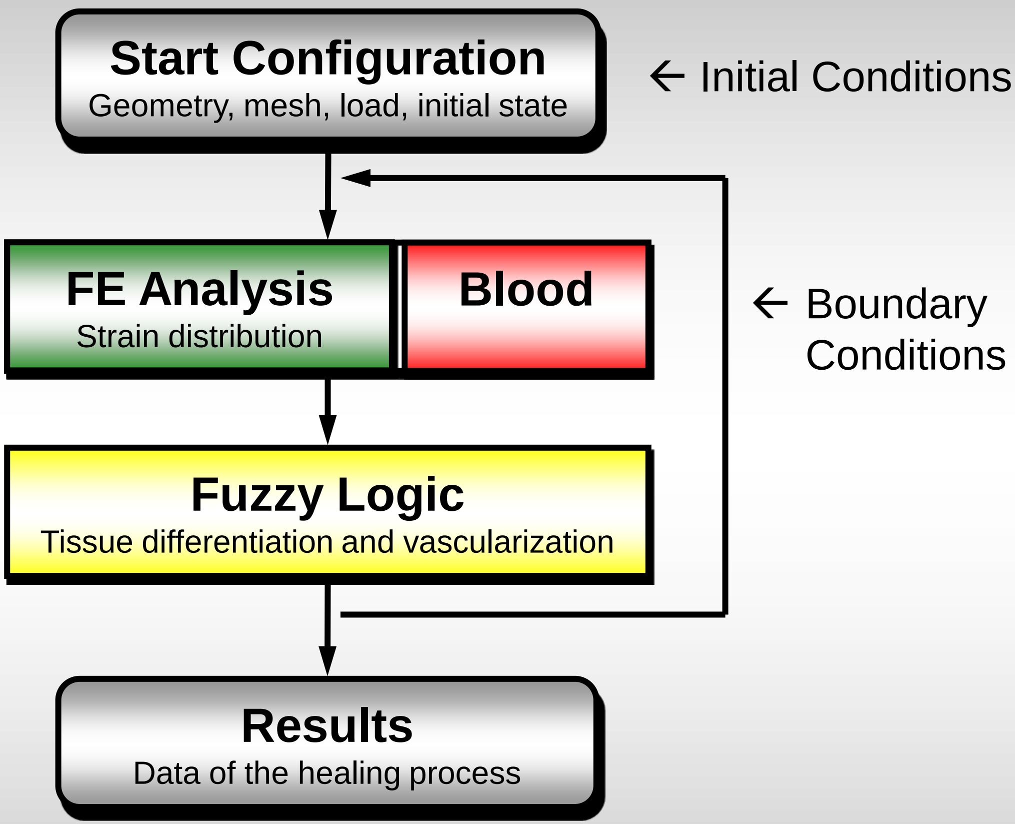
$$\underline{C}(\underline{x}, t) := \begin{bmatrix} \text{Blood perfusion} \\ \text{Cartilage concentration} \\ \text{Bone concentration} \end{bmatrix} = \begin{bmatrix} c_{\text{perf}}(\underline{x}, t) \\ c_{\text{cart}}(\underline{x}, t) \\ c_{\text{bone}}(\underline{x}, t) \end{bmatrix}$$

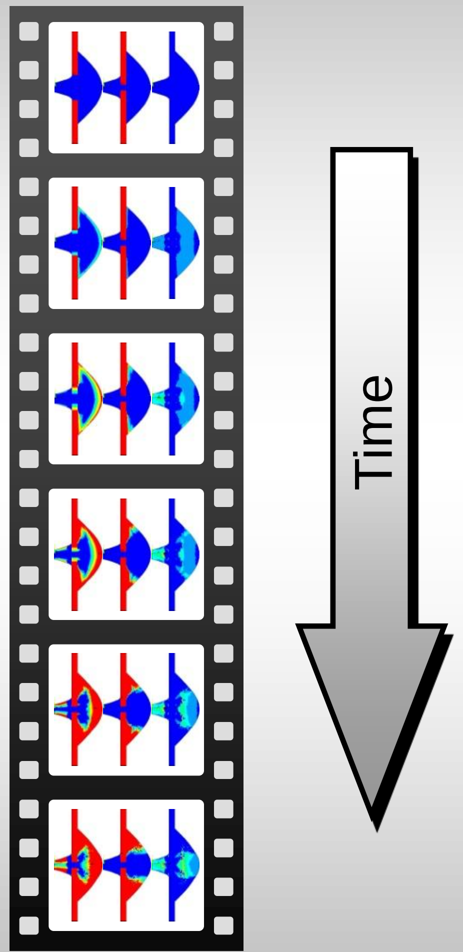
Rule of Mixture

$$E_{\text{el}} = \sum_{\text{tiss}} E_{\text{tiss}}\, c_{\text{el,tiss}}^{3}.$$

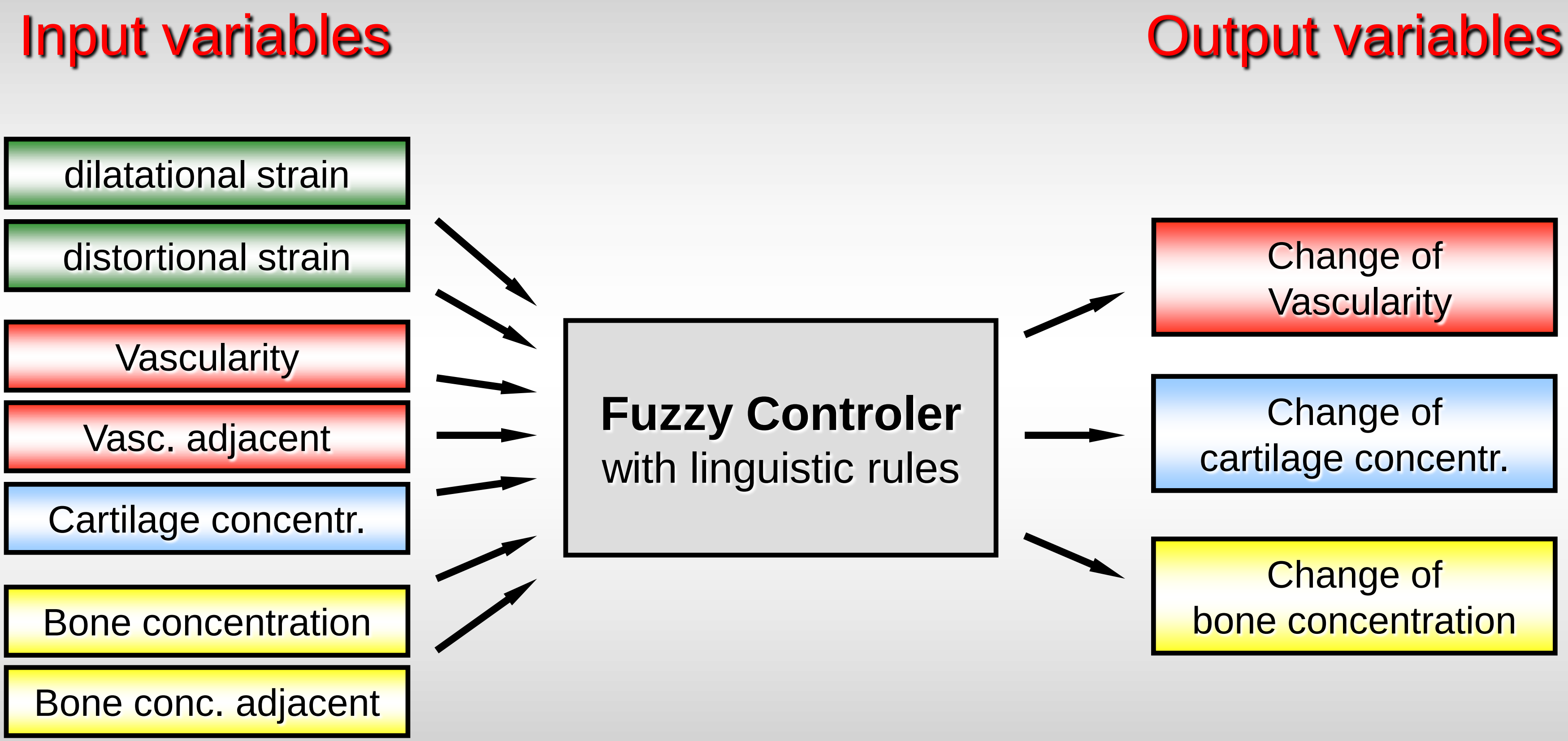
$$\nu_{\text{el}} = \sum_{\text{tiss}} \nu_{\text{tiss}}\, c_{\text{el,tiss}}.$$

Methods: Iterative Algorithm

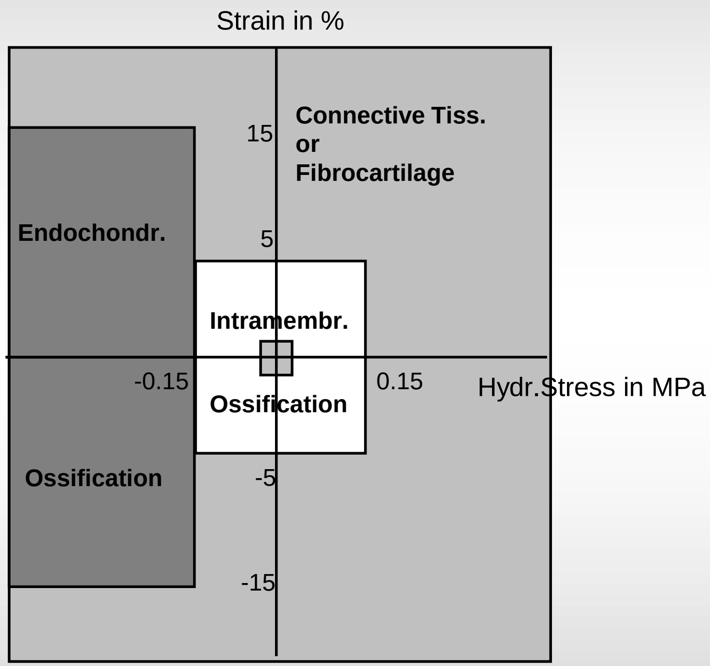




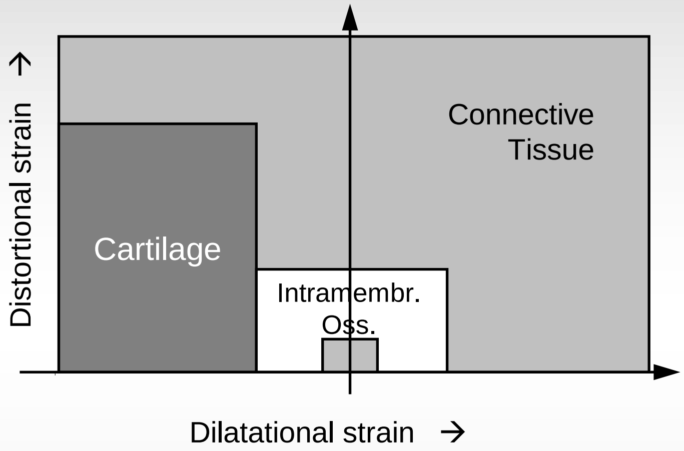
Methods: Tissue Healing with Fuzzy Logic



Tissue Transformation Function



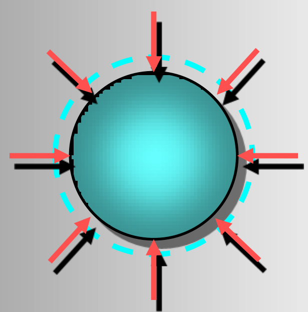
Claes and Heigele 99



Simon et al. 2011

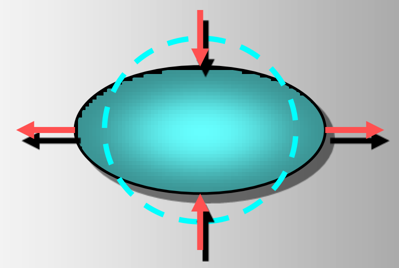
Methods: Mechanical Stimuli

Two invariants of the strain tensor



pure dilatational strain

In other studies:
- hydrostatic pressure
- octahedral (normal) stress
- fluid pressure / flow



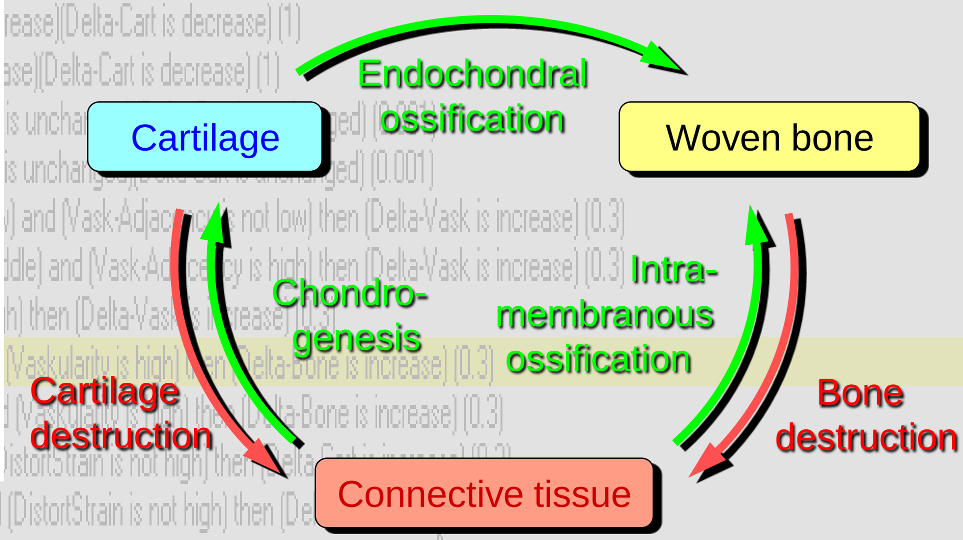
pure distortional strain

In other studies:
- octahedral shear stress
- von Mises equivalent stress
- von Mises equivalent strain

Methods: Fuzzy Rules

Processes:

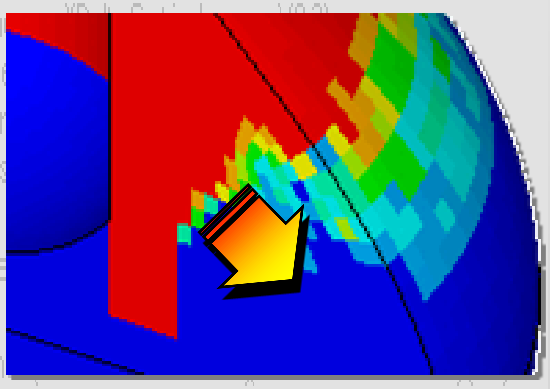
- Angiogenesis
- Intramembranous ossification
- Chondrogenesis
- Cartilage calcification
- Endochondral ossification
- Tissue destruction



Example: Rule #3, Intramembranous ossification:

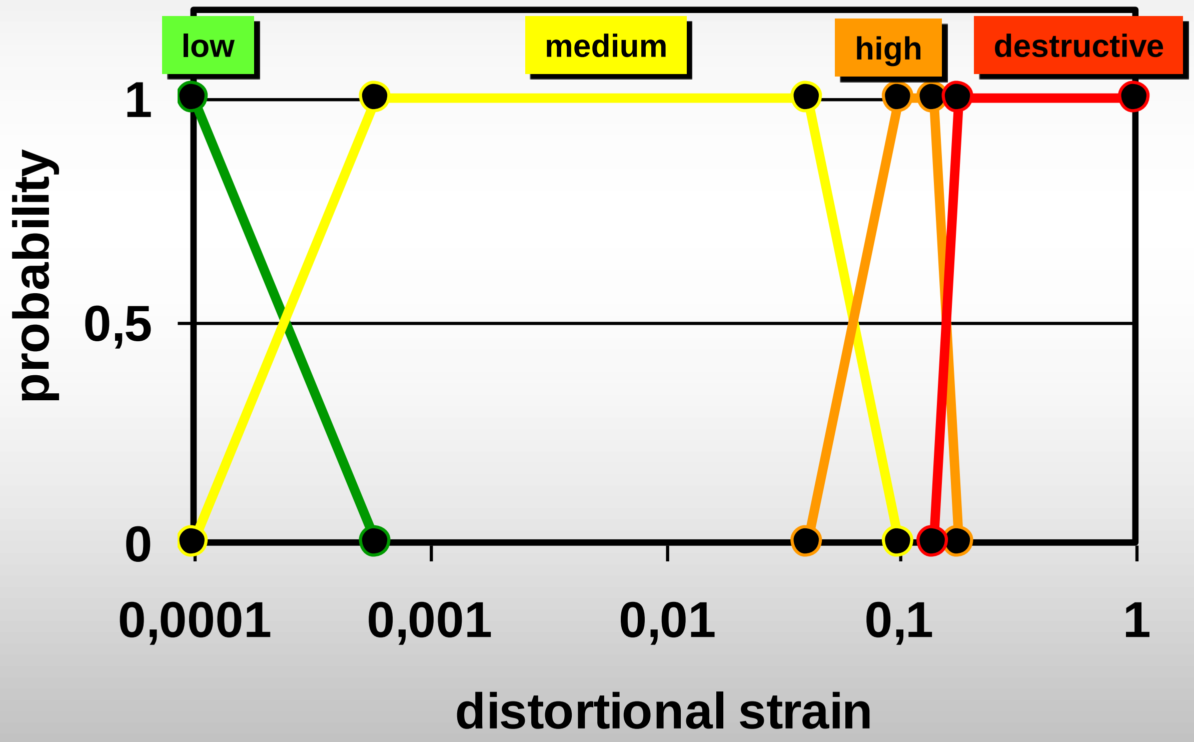
IF *(blood supply is high) AND (bone concentration adjacent is high) AND (dilatational strain is neg. low or pos. low) AND (distortional strain is low)*

THEN *(bone concentration has to increase)*

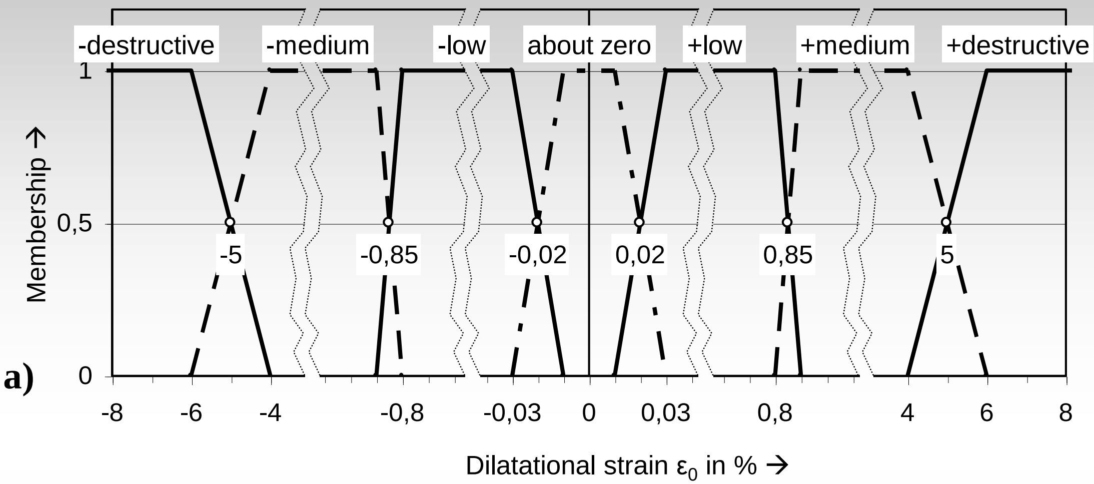


Methods: Membership Functions

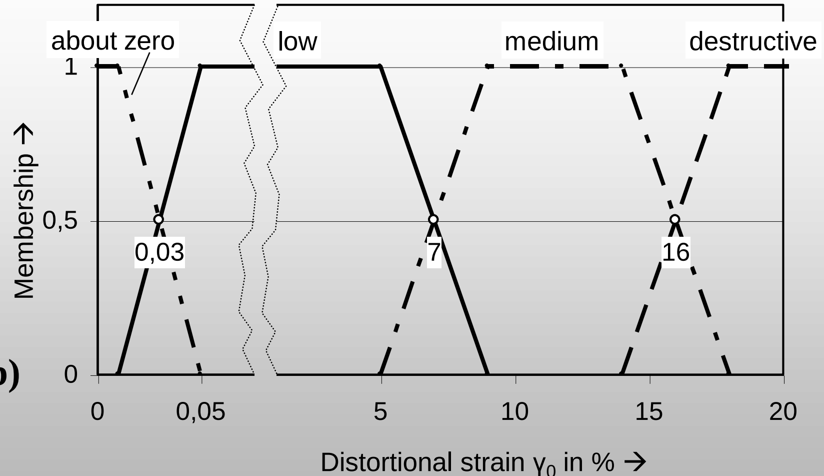
Example: One of 10 membership functions



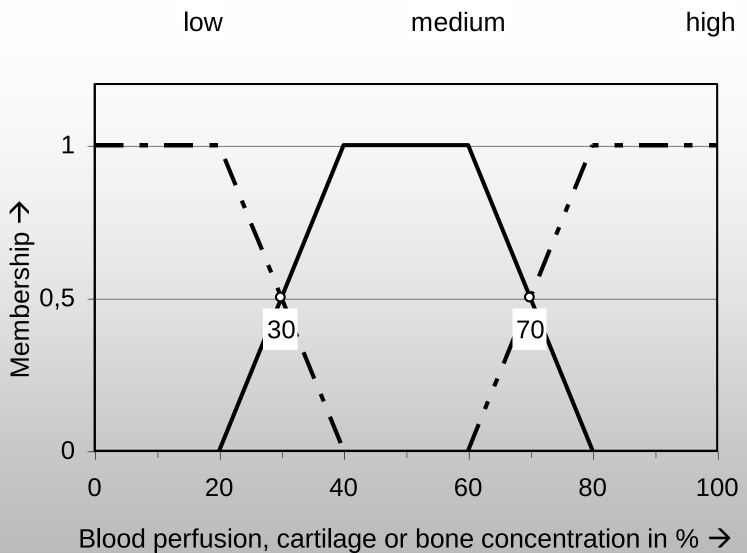
Zugehörigkeitsfunktionen für Eingangsgrößen



a)

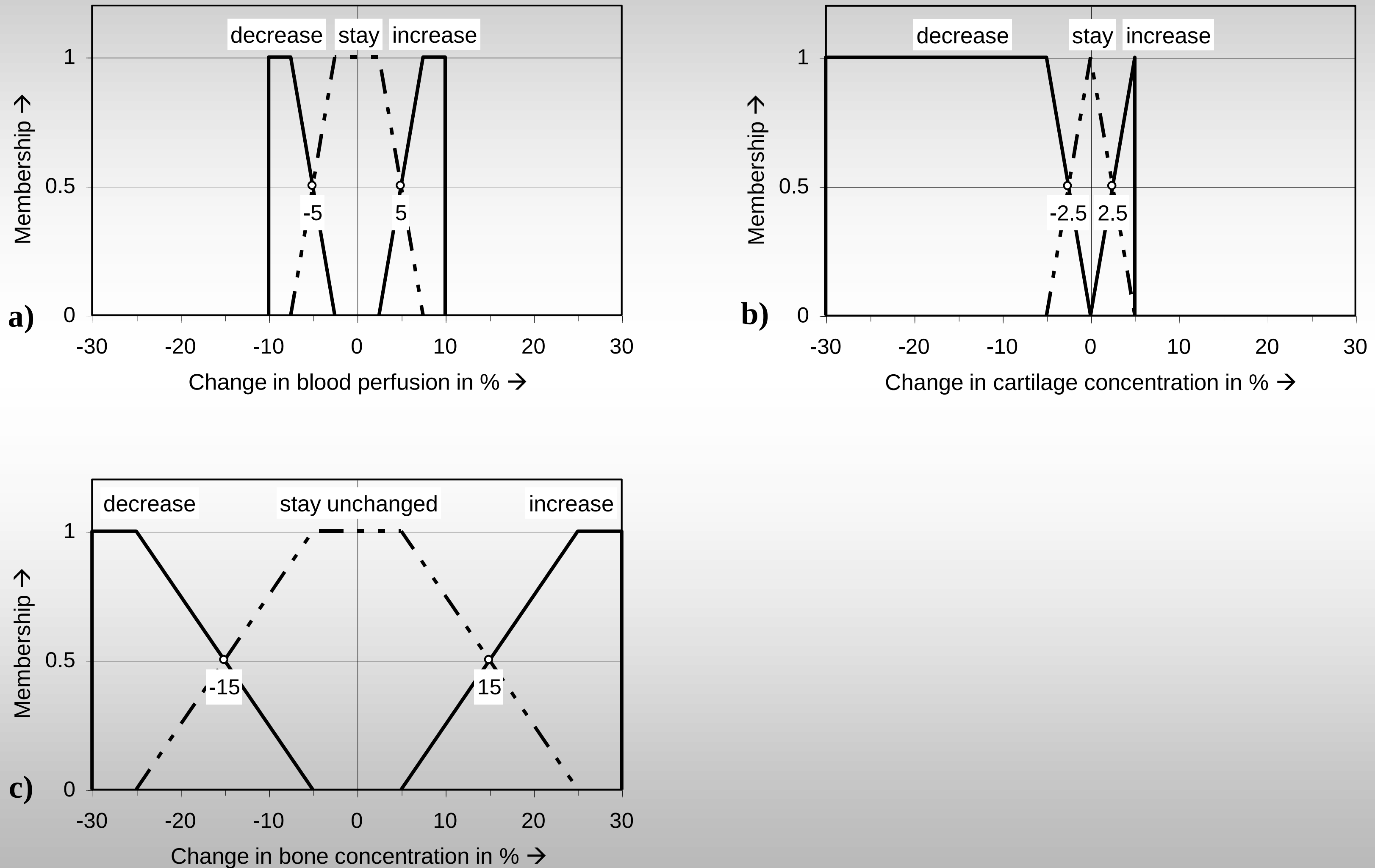


b)



c)

Zugehörigkeitsfunktionen für Ausgangsgrößen

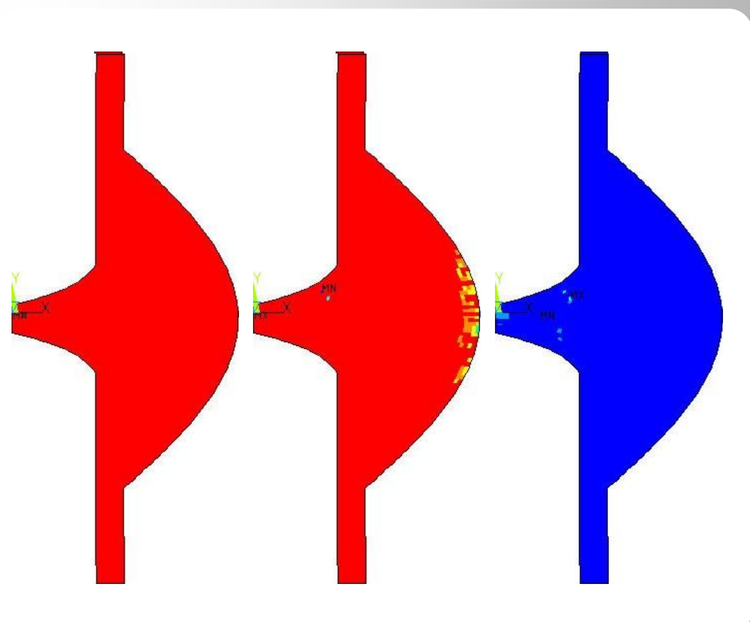


Results

Case A

Small initial movement 0.25 mm

Blood Bone Cartilage

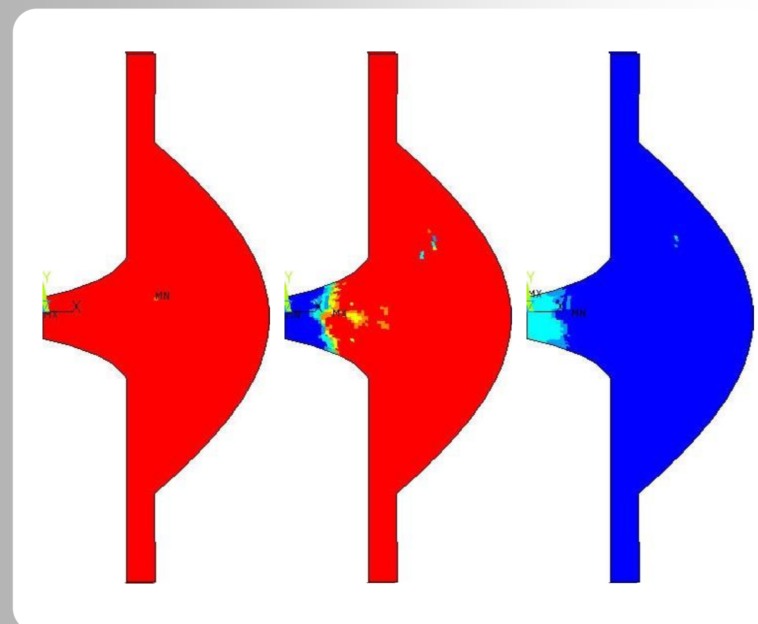


Day 63

Case B

Large initial movement 1.25 mm

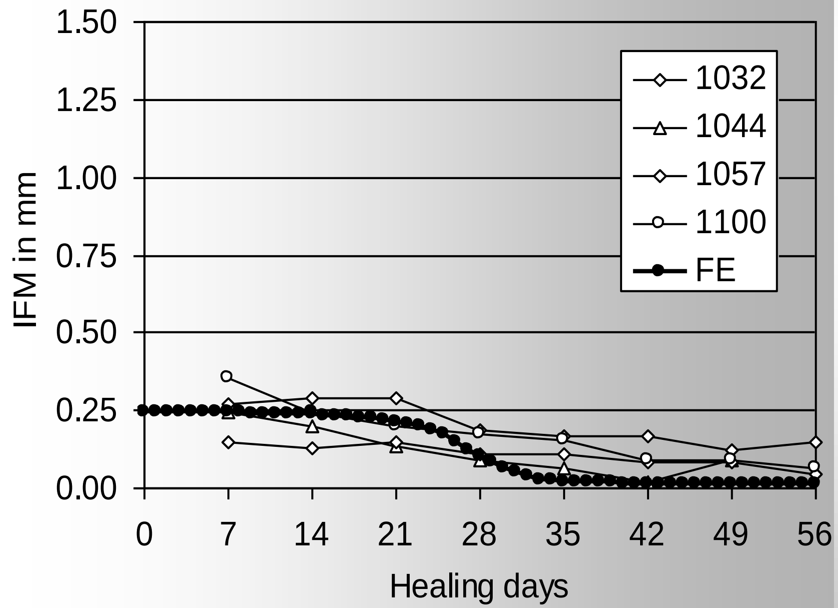
Blood Bone Cartilage



Results: Interfragmentary Movement

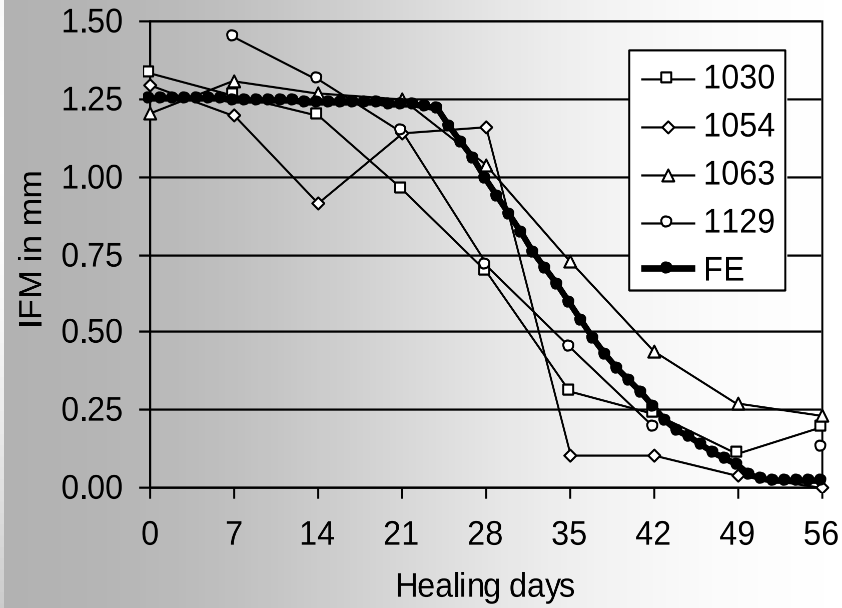
Case A

Small initial movement 0.25 mm



Case B

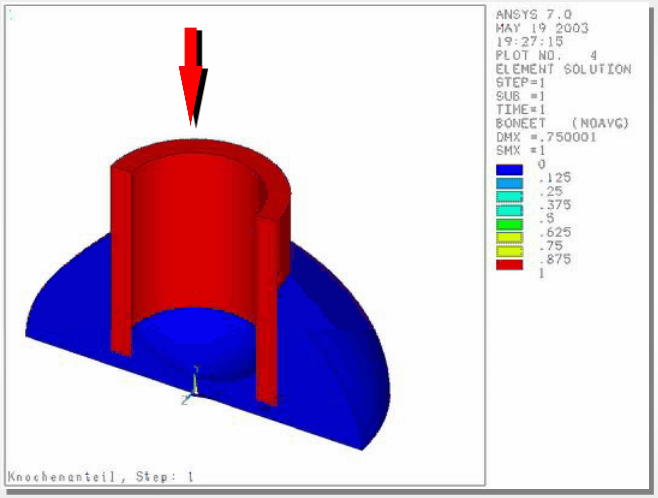
Large initial movement 1.25 mm



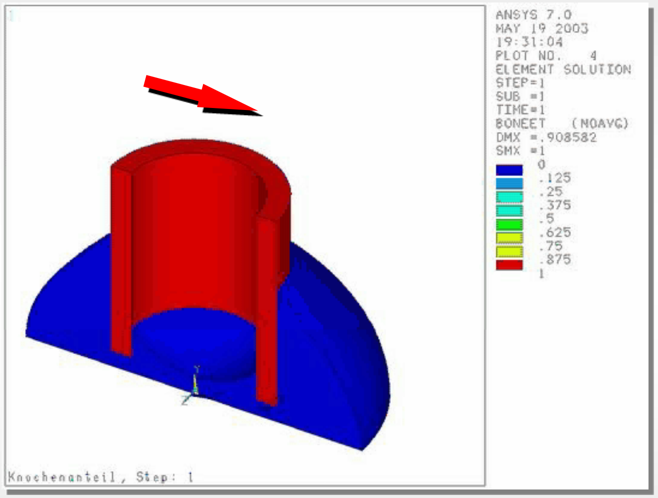
Animal experiments: Claes, et al. 97

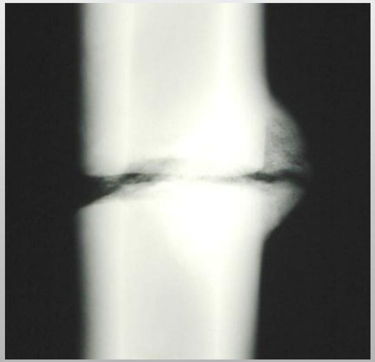
Simulation of 3D bone healing under axial and shear movement

Axial movement

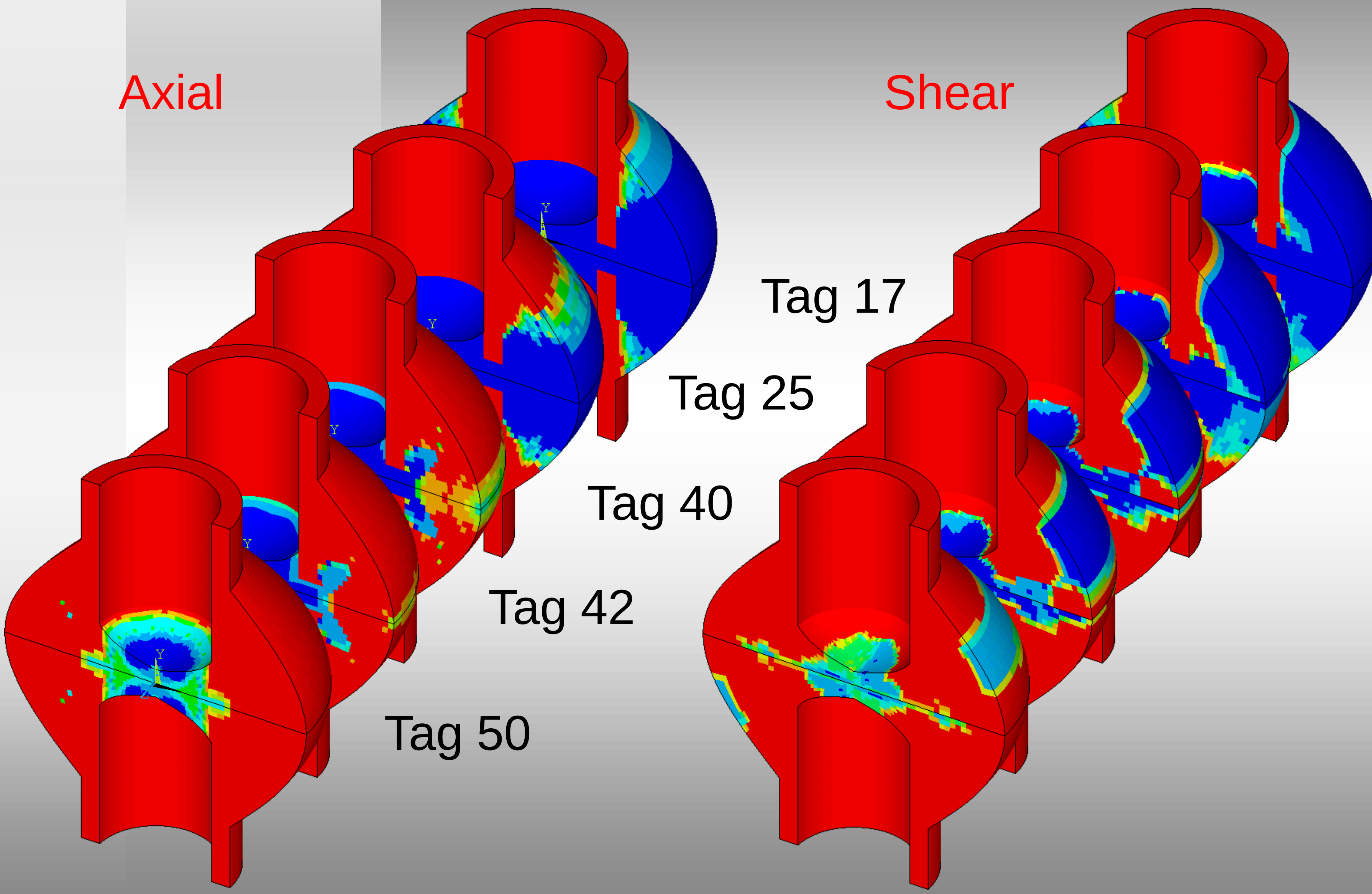


Shear movement

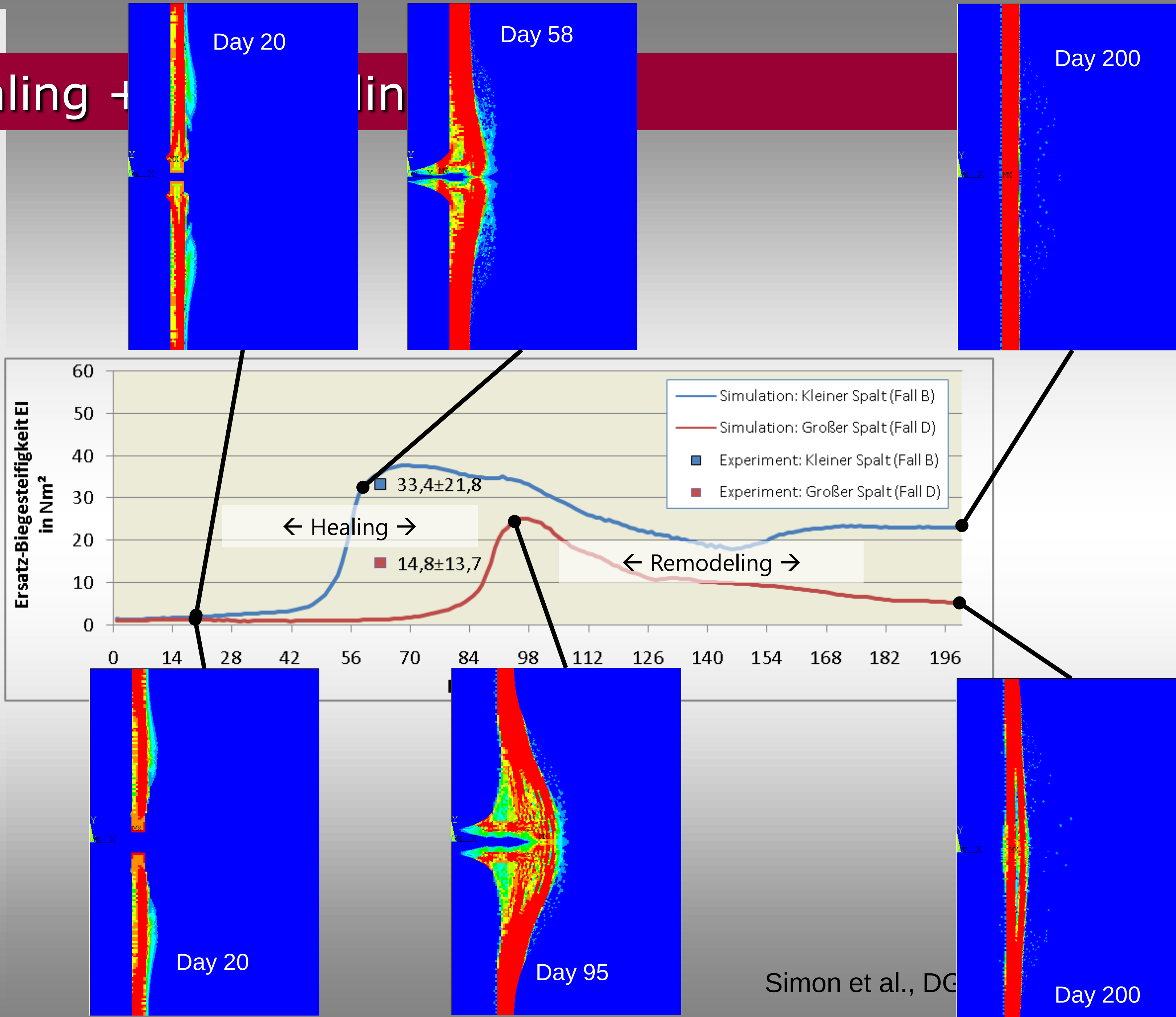




3D Stiffness Directions

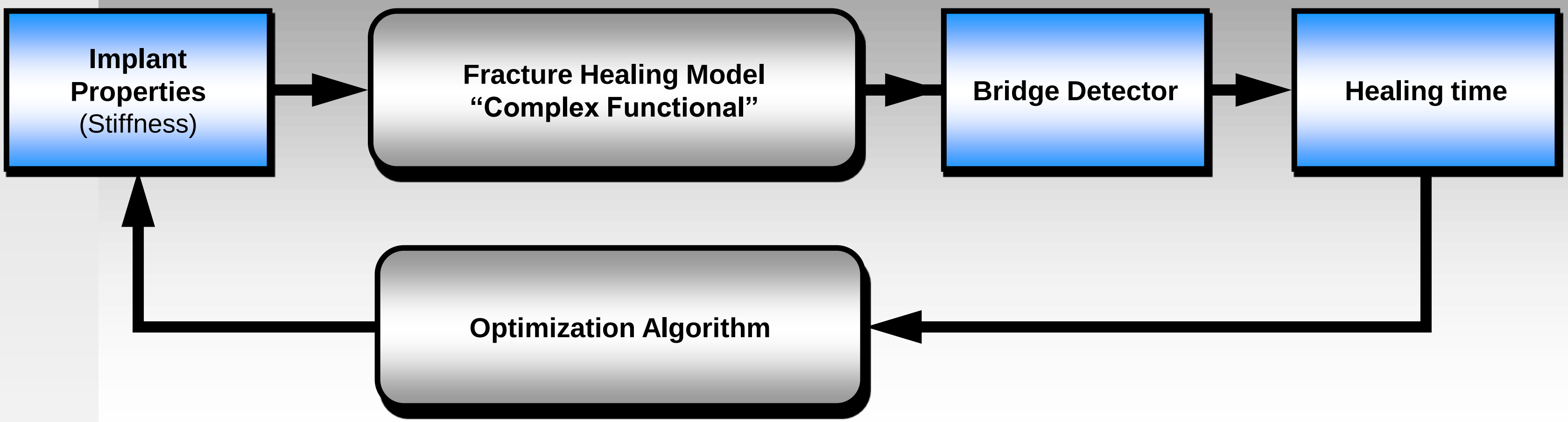


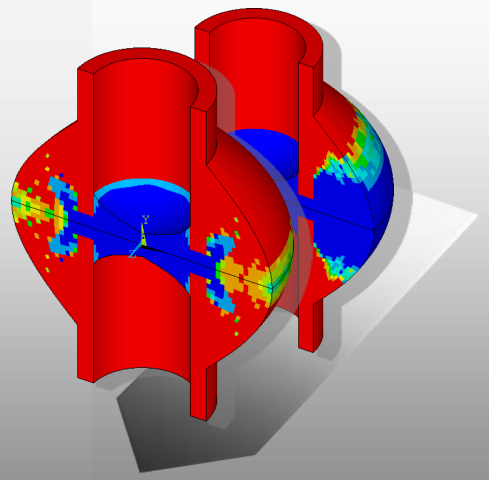
Healing + … lin

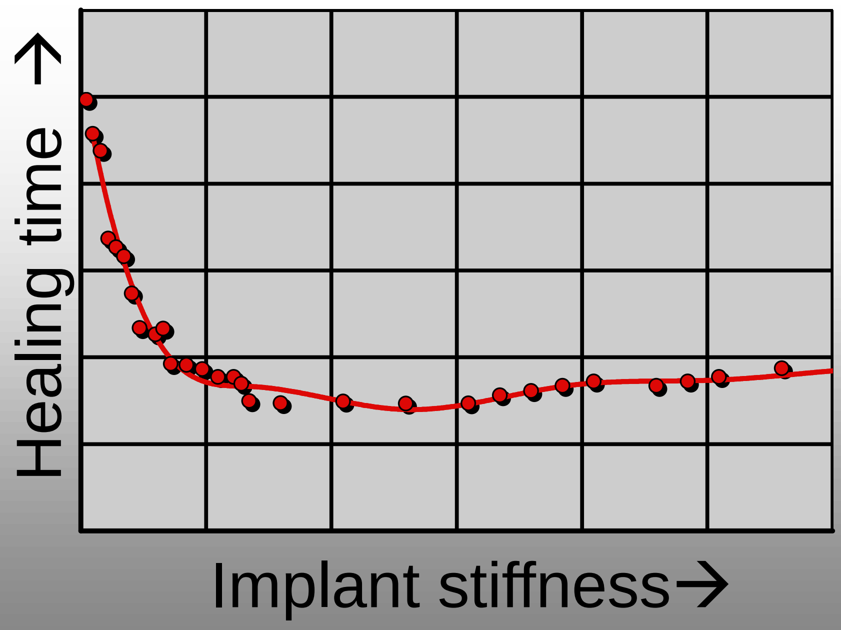


Simon et al., DG

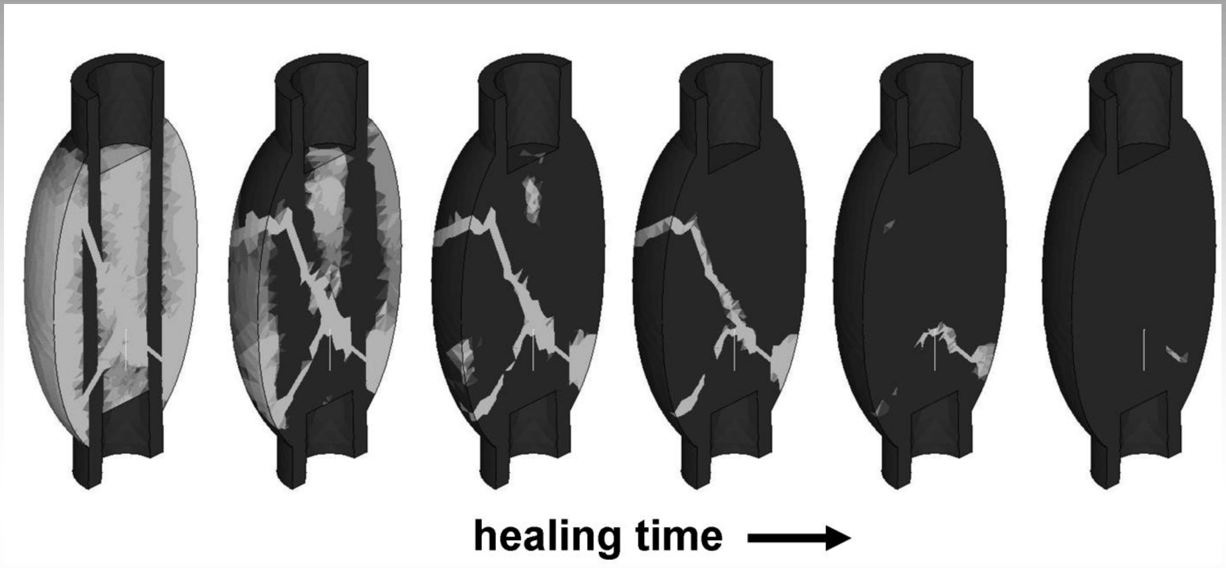
Optimal Implant Stiffness?

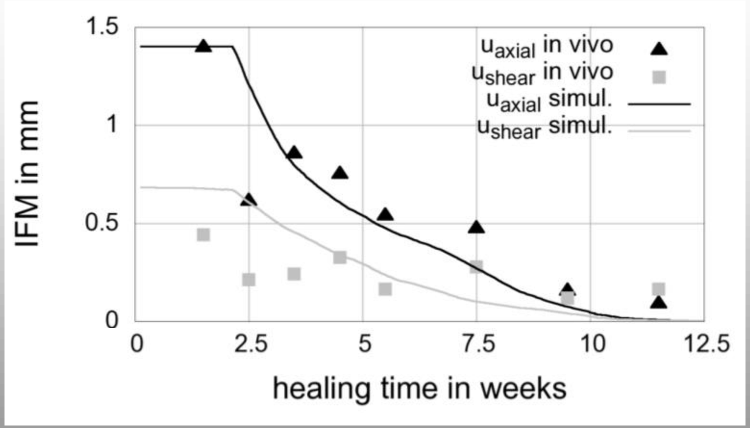






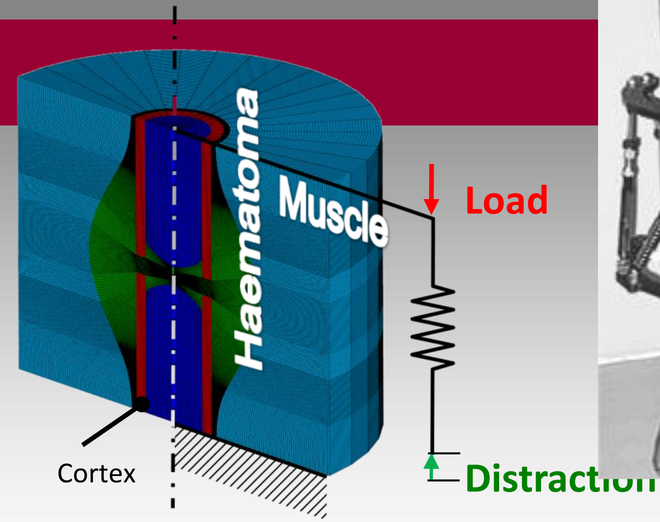
Complex Fracture Geometry

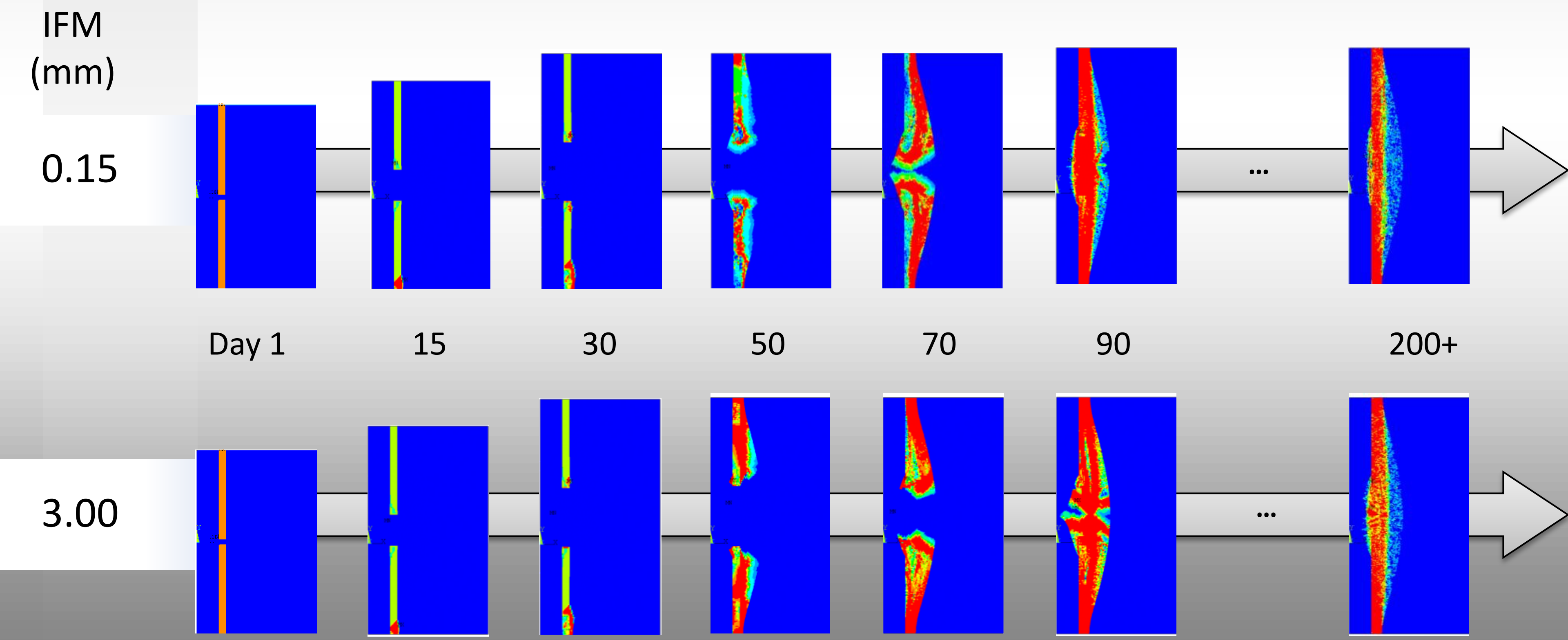




Wehner et al., Clin Biom 2010

Distraction Osteogenesis





Mechanical Basics

1.3 Variables, Dimensions and Units

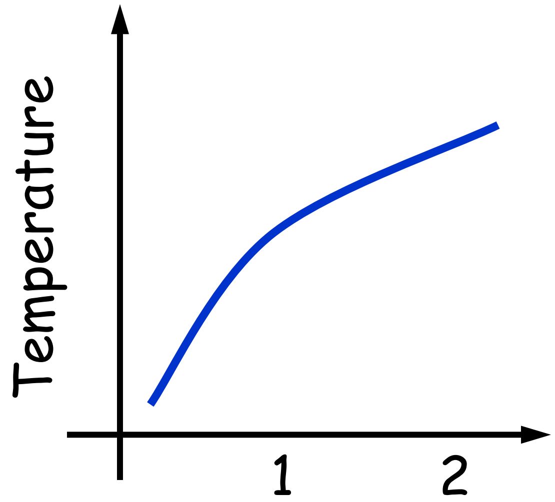
Standard: ISO 31, DIN 1313

Variable $=$ Number $\cdot$ Unit

Length L $= 2 \cdot \text{m} = 2\ \text{m}$

{Variable} = Number

[Variable] = Unit



~~Length L [m]~~

Length L / m

Length L in m →

Three mechanical SI-Units:

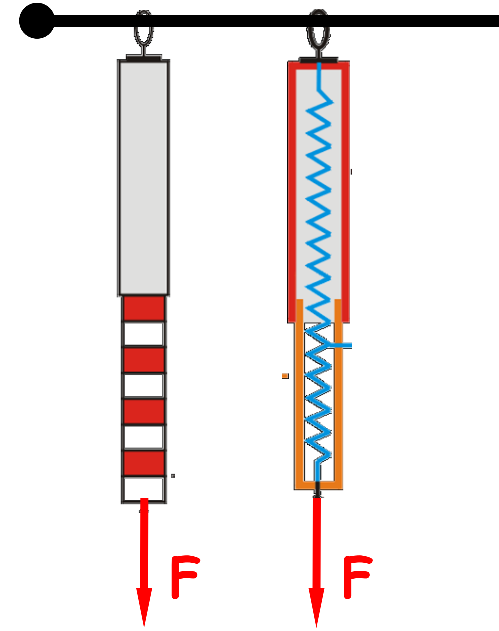
m (Meter)

kg (Kilogram)

s (Seconds)

2 STATICS OF RIGID BODIES

2.1 Force



- We all believe to know what a *force* is.
- But, *force* is an invention not a discovery!
- … it can not be measured directly.

Newton's 2nd Law [Axiom]:

Force = Mass times Acceleration or $F = m \cdot a$

Note to Remember:

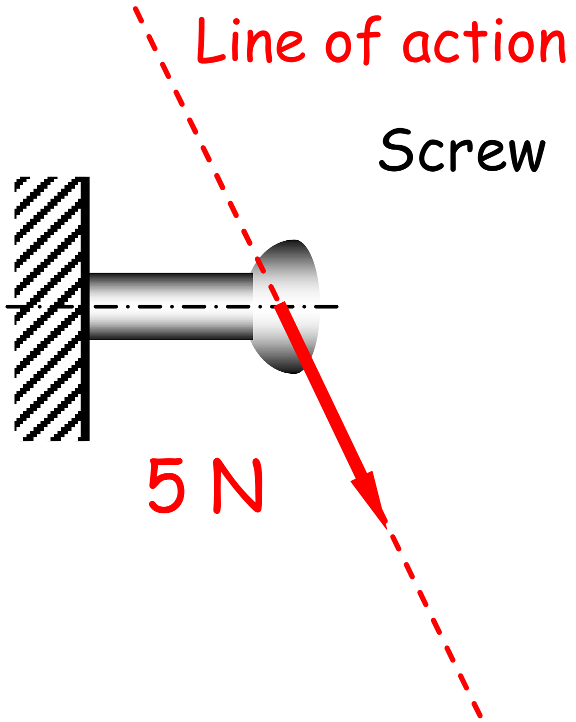
„A force is the cause of acceleration or deformation of a body"

Representation of Forces

... with arrows

Forces are Vectors with

- **Magnitude**
- **Direction**
- **Sense of Direction**



Units of Force

Newton

N = kg·m/s²

$N = kg \cdot m/s^2$

$F_G = m \cdot g = 0{,}1\ kg \cdot 9{,}81\ m/s^2$

$= 0{,}981\ kg\ m/s^2$

$\approx 1\ N$

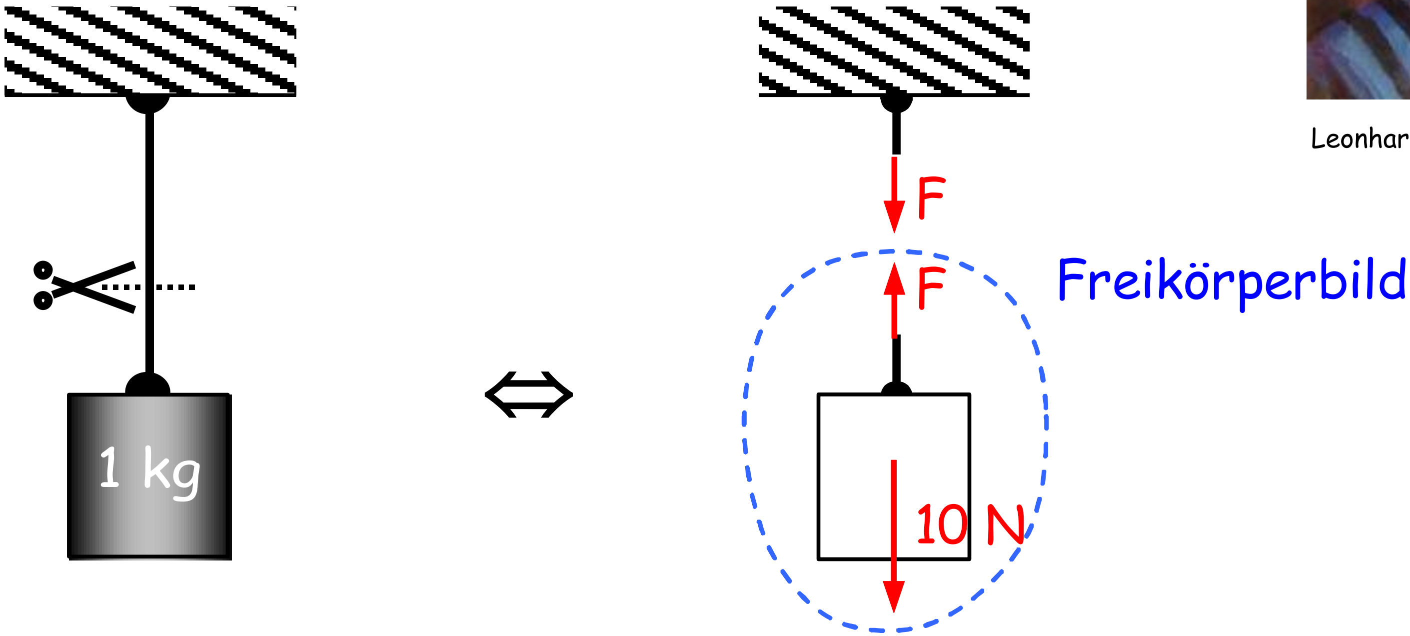
Note to Remember:

1 Newton ≈ Weight of a bar of chocolate (100 g)

2.2 Method of Sections (Euler) [Schnittprinzip]

Free-Body Diagramm (FBD) [Freikörper-Bild]

Leonhard Euler, 1707 - 1783

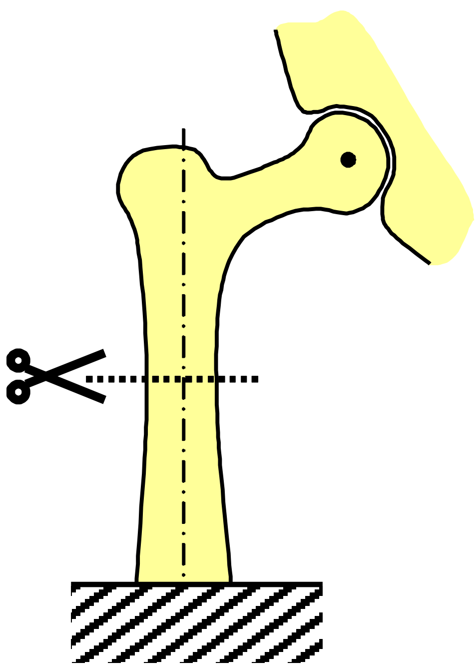


Note to Remember:

First, cut the system, then include forces and moments.

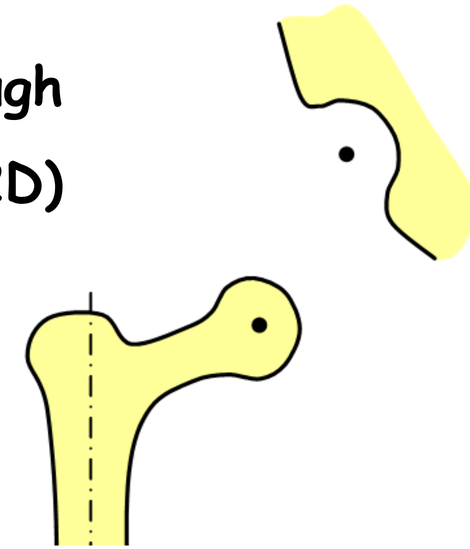
Free-body diagram = <u>completely</u> isolated part.

<u>2.2 Method of Sections</u>

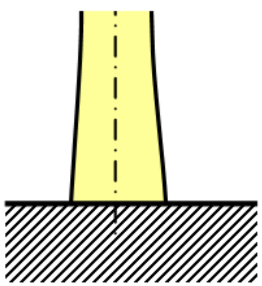


⇔

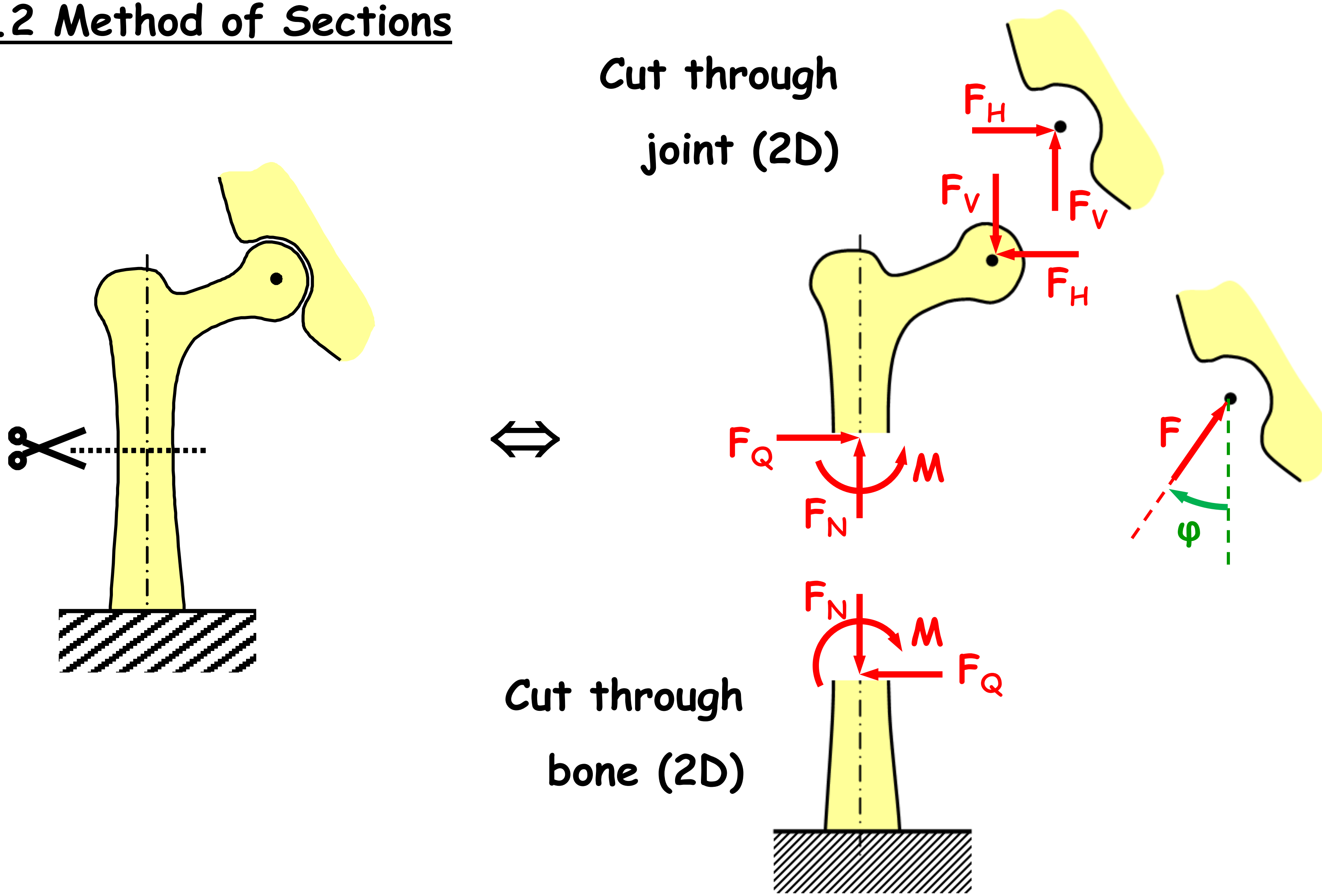
Cut through joint (2D)



Cut through bone (2D)

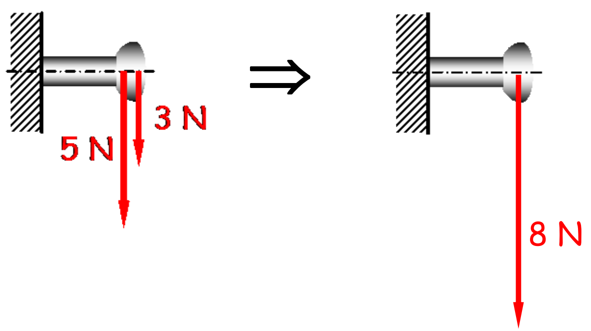


2.2 Method of Sections

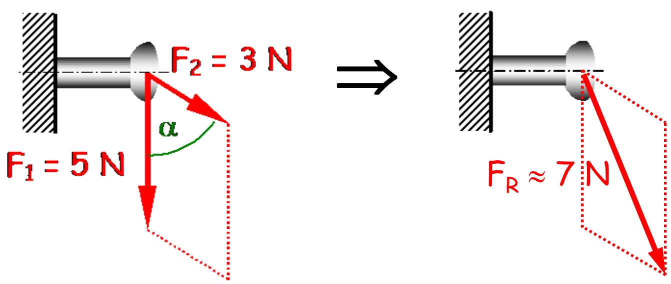


2.3 Combining and Decomposing Forces

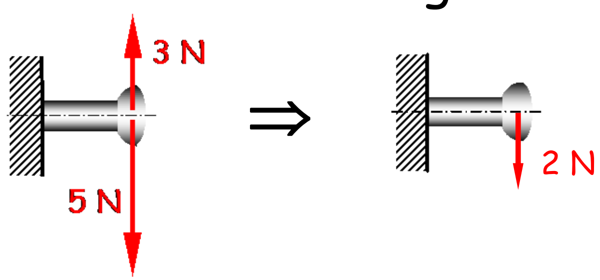
Summation of Magnitudes



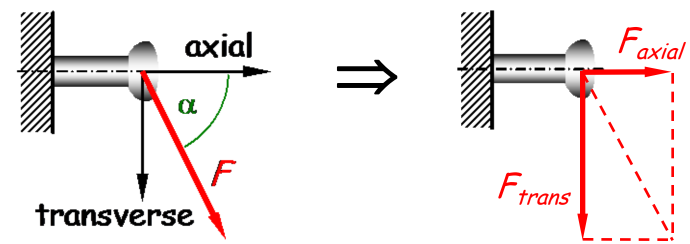
Vector Addition



Subtraction of Magnitudes

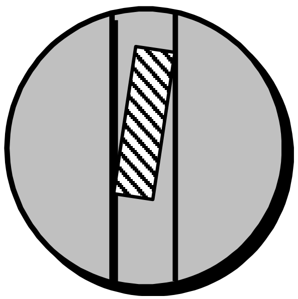


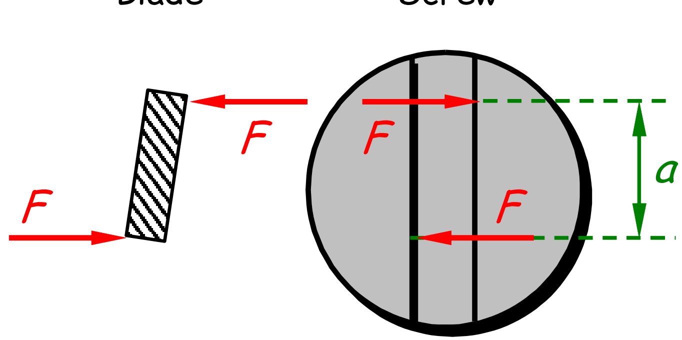
Decomposition into Components



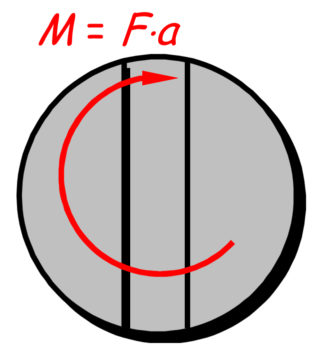
2.4 The Moment [Das Moment]

Slotted screw with screwdriver blade





Force Couples (F, a)



Moment M

Note to remember:

The moment $M = F \cdot a$ is equivalent to a force couple (F, a).

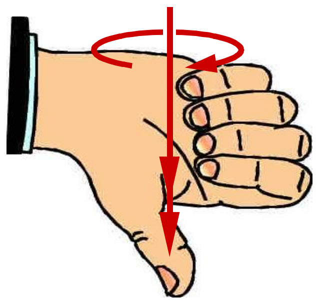
A moment is the cause for angular acceleration or angular deformation (Torsion, Bending) of a body.

Units for Moment

Newton-Meter

$N \cdot m \ = \ kg \cdot m^2/s^2$

Rechte-Hand-Regel:

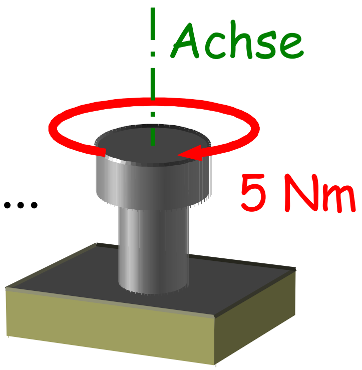


Representation of Moments

... with rotation arrows or double arrows

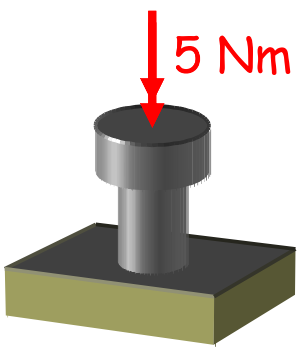
Moments are Vectors with ...

- **Magnitude**
- **Direction**
- **Sense of Direction**



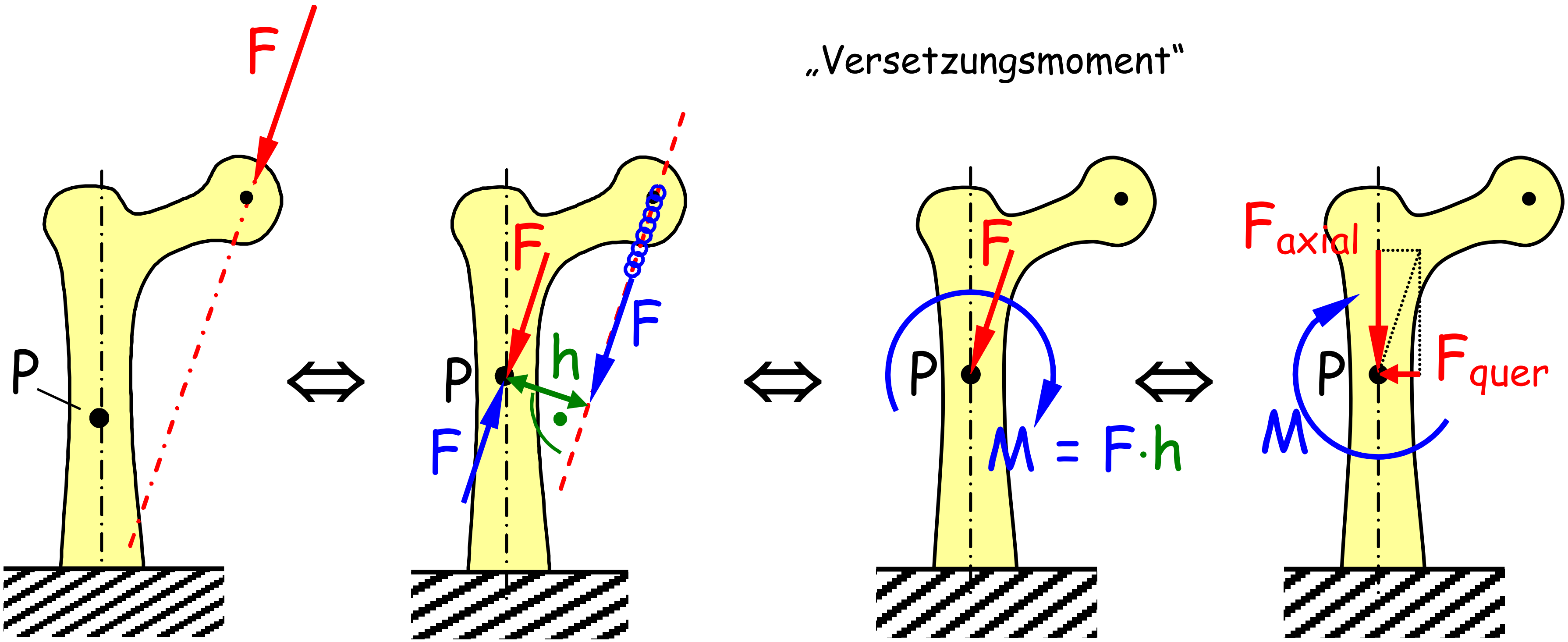
Drehpfeil

oder



Doppelpfeil

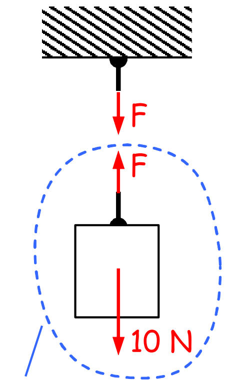
2.5 Moment of a Force about a Point [Versetzungsmoment]



Note to Remember:

Moment = Force times lever-arm

2.7 Static Equilibrium



Free-body diagram (FBD)

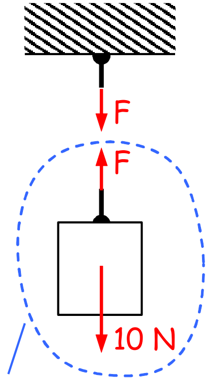
> **Important:**
>
> Free-body diagram (FBD) first, then equilibrium!

For 2D Problems max. 3 equations for each FBD:

The sum of all forces in *x*-direction equals zero:	$F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0$
The sum of all forces in *y*-direction equals zero:	$F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0$
The sum of Moments with respect to P equals zero:	$M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0$

(For 3D Problems max. 6 equations for each FBD)

2.7 Static Equilibrium



Important:

Free-body diagram (FBD) first, then equilibrium!

3 equations of equilibrium for each FBD in 2D:

$$\text{Sum of all forces in x-direction:} \quad F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0,$$

$$\text{Sum of all forces in y-direction:} \quad F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0,$$

$$\text{Sum of all moments w.resp.to P:} \quad M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0.$$

- Force EEs can be substituted by moment EEs
- 3 moment reference points should not lie on one line

6 equlibrium equations for one FBD in 3D:

$$\text{Summe aller Kräfte in x-Richtung:} \quad \sum_i F_{ix} \overset{!}{=} 0,$$

$$\text{Summe aller Kräfte in y-Richtung:} \quad \sum_i F_{iy} \overset{!}{=} 0,$$

$$\text{Summe aller Kräfte in z-Richtung:} \quad \sum_i F_{iz} \overset{!}{=} 0,$$

$$\text{Summe aller Momente um x-Achse bezüglich Punkt P:} \quad \sum_i M_{ix}^{P} \overset{!}{=} 0.$$

$$\text{Summe aller Momente um y-Achse bezüglich Punkt Q:} \quad \sum_i M_{iy}^{Q} \overset{!}{=} 0.$$

$$\text{Summe aller Momente um z-Achse bezüglich Punkt R:} \quad \sum_i M_{iz}^{R} \overset{!}{=} 0.$$

- Force EEs can be substituted by moment EEs
- Max. 2 moment axis parallel to each other
- Determinant of coef. matrix not zero

2.8 Recipe for Solving Problems in Statics

Step 1: Model building. Generate a simplified replacement model (diagram with geometry, forces, constraints).

Step 2: Cutting, Free-body diagram. Cut system and develop free-body diagrams. Include forces and moments at cut, as well as weight.

Step 3: Equilibrium equations. Write the force- and moment equilibrium equations (only for free-body diagrams).

Step 4: Solve the equations. One can only solve for as many unknowns as equations, at most.

Step 5: Display results, explain, confirm with experimental comparisons. Are the results reasonable?