Computational Biomechanics 2018

Lecture 04:
Material Laws for Biological Tissues

Ulli Simon, Martin Pietsch, Lucas Engelhardt

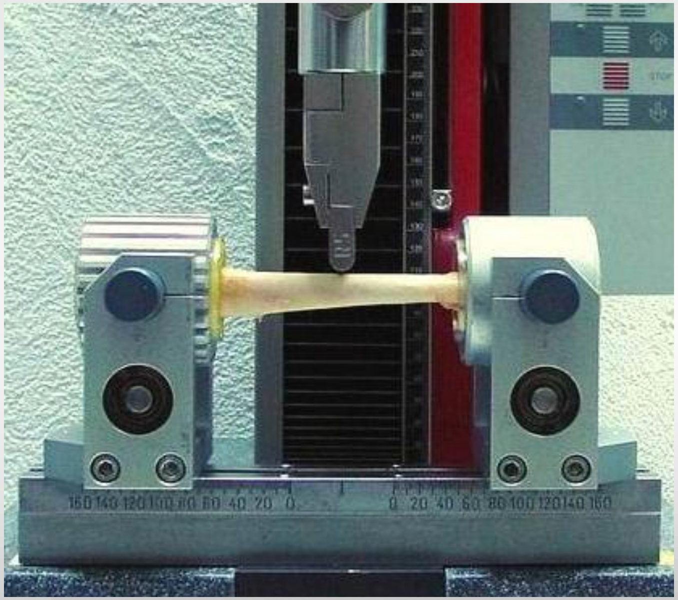
Scientific Computing Centre Ulm, UZWR
Ulm University

Material Laws
for Biological Tissues

Characterizing Mechanical Properties

Structural Properties

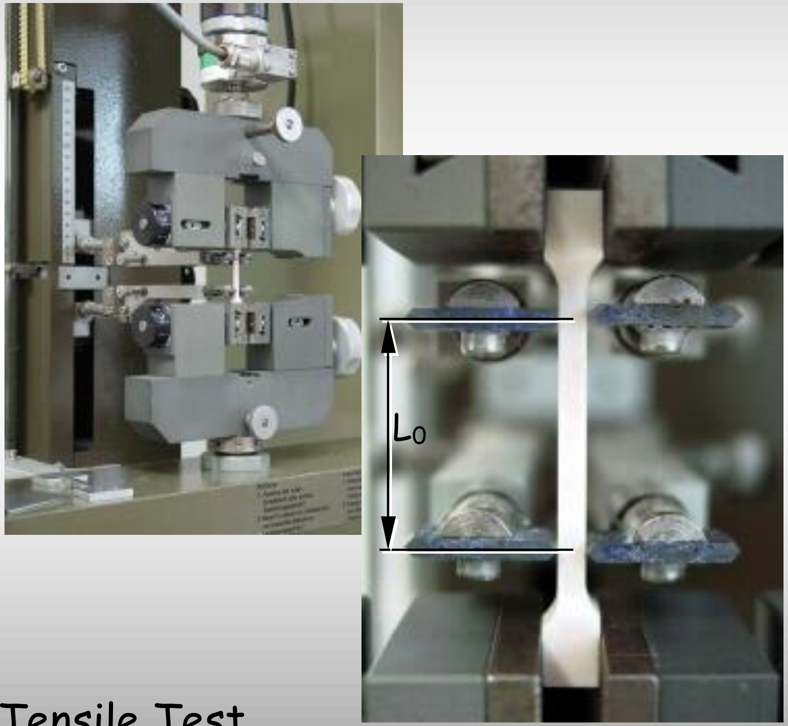
(Organ Properties)



3-Point Bending
of a
bone specimen

Material Properties

(Tissue Properties)

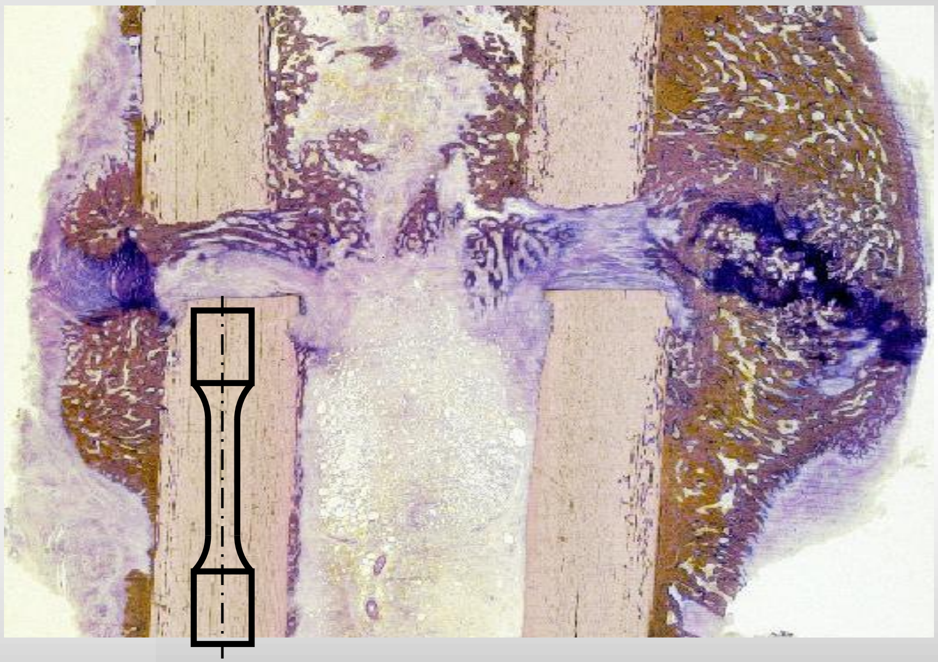


Tensile Test
of a
standardized Specimen

Mechanical Properties

	Organ Properties	Tissue Properties
Elastic	Different stiffnesses: load/deformation	Young's Modulus Poisson's Ratio Shear Modulus Compressive Modulus
Ultimate	Ultimate forces, moments, displacements: tension, compression, bending, torsion	Ultimate stresses, strains: tensile, compressive, shear at fracture at end of elastic region, at start of yielding

Example: Fracture Callus



Haematoma	
Granulation tissue	0,01 ... 3 MPa
Fibrous tissue	
Cartilage	0,4 … 500 MPa
Woven bone	400 … 6000 MPa
Cortical bone	10 ... 20 GPa

Yamada 1970; Hori & Lewis 1982; Davy & Connolly 1982; Proctor et al. 1989; Mente & Lewis 1994; Rho et al. 1995; Augat et al. 1998; Tanck et al. 1999

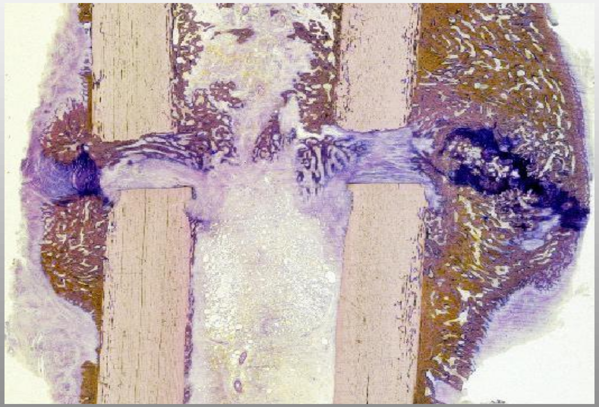
Overview

Simplest Material Law:

- Linear-elastic, isotropic (Hook's Law)

More complex Laws:

- Nonlinear
- Hyperelastic (nonlinear elastic + large deformations)
- Viscoelastic
- Non-elastic, plastic, yielding, hardening
- Anisotropic
- Fatigue, recovery
- Remodeling, healing



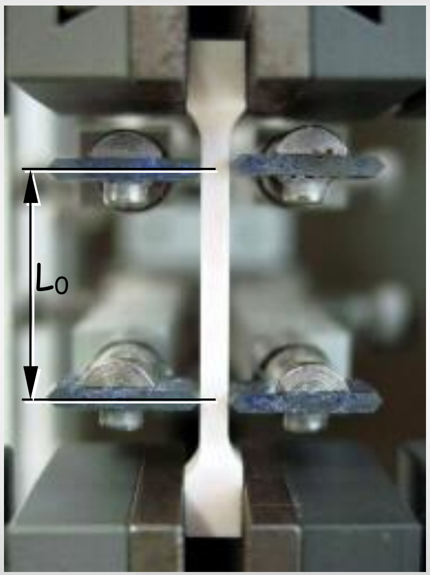
Overview

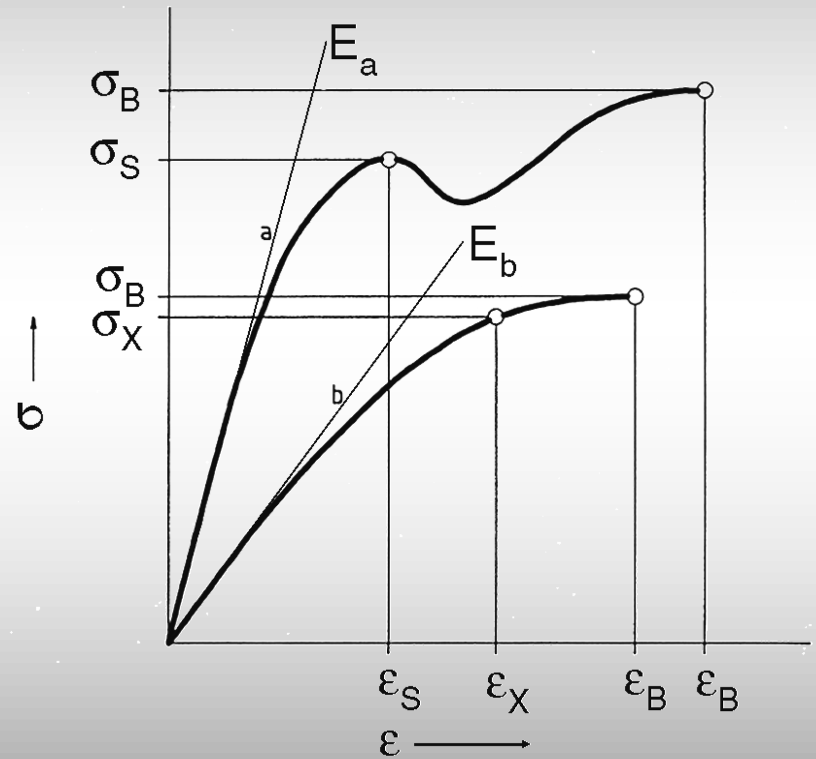
In principal:

Biological Tissues know all of these bad things.

They often combine the bad things

Stress-strain curve





σ_B	*Ultimate stress*
σ_S	*Yield point*
ε_B	*Ultimate strain*
ε_S	*Strain at yoild point*
E	Young's modulus

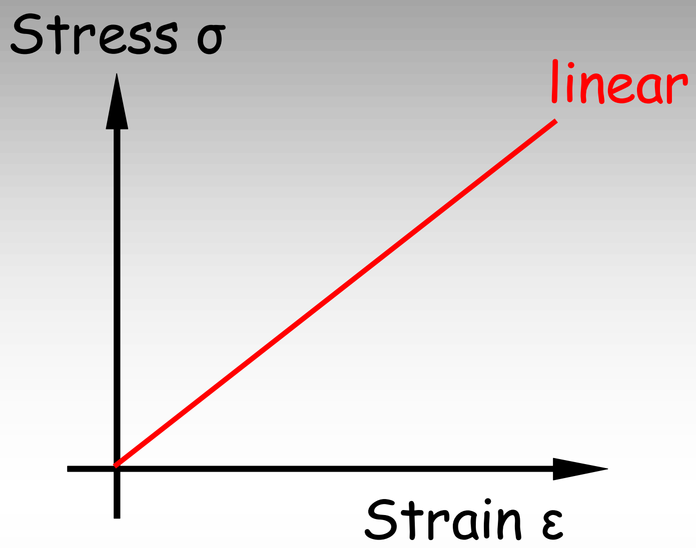
Linear Elastic Law

Linear stress-strain relation

$$\sigma = E \cdot \varepsilon$$ Hook's Law

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{E}}}} \cdot \underline{\underline{\varepsilon}}$$ 3D

$$\sigma_{ij} = E_{ijkl} \cdot \varepsilon_{kl}$$



- Full 3^4 material properties tensor of 4^{th} order (81)
- Equality of shear stresses (Boltzmann Continua) and strains (36)
- Reciprocity Theorem from Maxwell* → fully anisotropic (21)
- Orthotropic (9)
- Transverse Isotropic (5)
- Isotropy (2)

*) also known as Betti's theorem or Maxwell-Betti reciprocal work theorem

Linear Elastic Law

$$\underline{\sigma} = \underline{\underline{E}} \cdot \underline{\varepsilon}$$

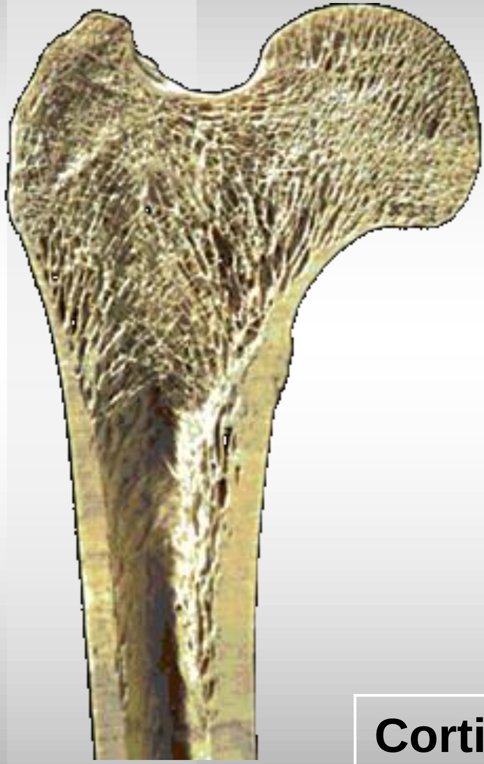
$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \frac{E}{(1+\nu)\cdot(1-2\nu)} \cdot \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ & (1-\nu) & \nu & 0 & 0 & 0 \\ & & (1-\nu) & 0 & 0 & 0 \\ & & & \frac{(1-2\nu)}{2} & 0 & 0 \\ & & & & \frac{(1-2\nu)}{2} & 0 \\ \text{sym} & & & & & \frac{(1-2\nu)}{2} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

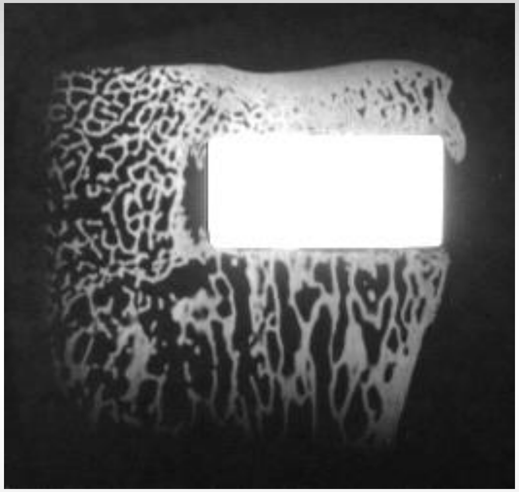
E	-	Young's modulus
ν	-	Poisson's ratio (0 ... 0.5)
G	-	Shear modulus
K	-	Bulk modulus
μ, λ	-	Lame Constants

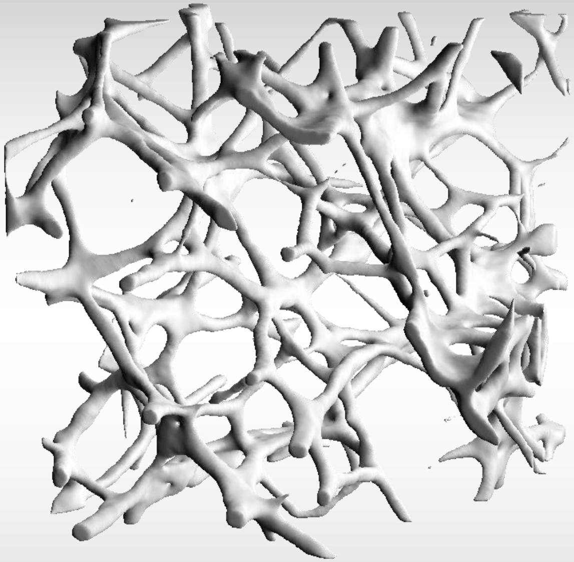
← 2 of these

Anisotropic Properties

→ **Bone tissue**







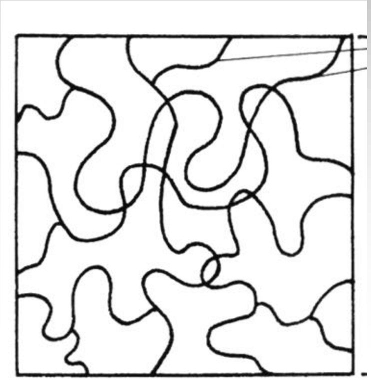
Corticale bone	Transverse isotropic (5)	Like Wood
Trabecular bone	Orthotropic (9)	3D Grid

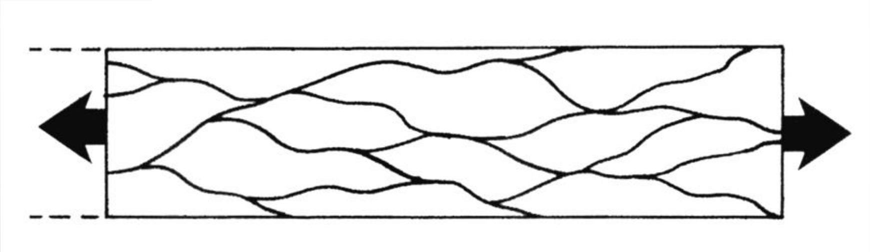
Anisotropic Properties

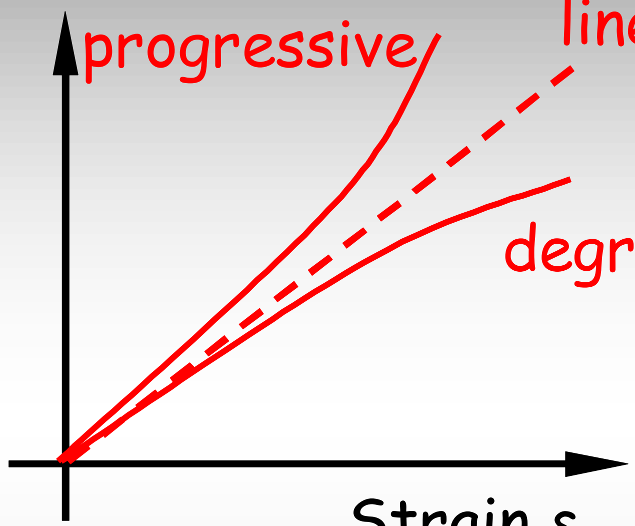
Material	E Moduli in MPa	Strength in MPa	Fracture strain in %
Spongy bone			
Vertebra	60 (male) 35 (female)	4,6 (male) 2,7 (female)	6
prox. Femur	240	2.7	2.8
Tibia	450	5...10	2
Bovine	200...2000	10	1,7...3,8
Ovine	400...1500	15	
Cortical bone			
Longitudinal	17000	200	2,5
Transversal	11500	130	

Nonlinear Elastic and Hyperelastic Law

→ soft, fibrous tissues







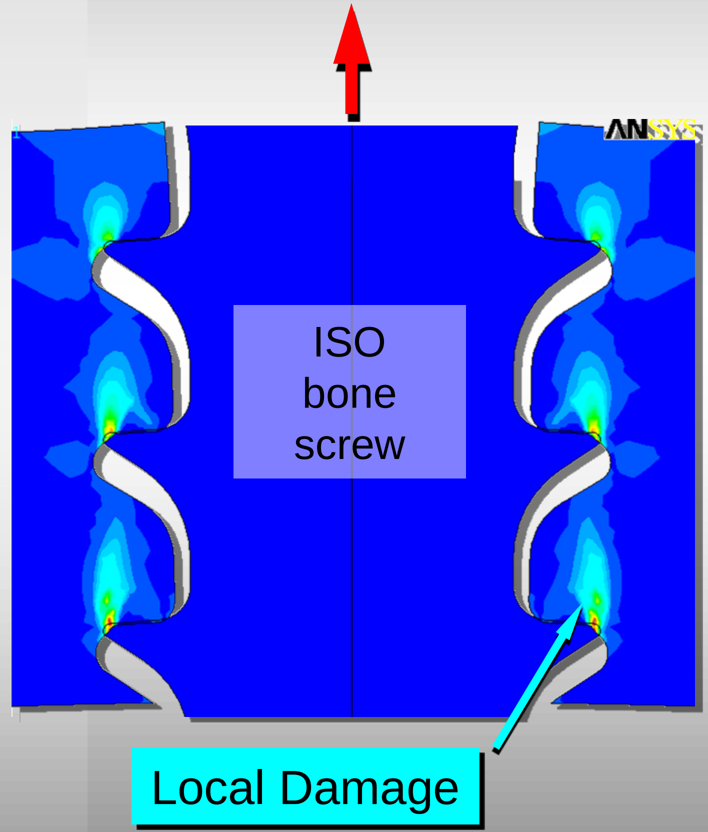
Hyperelastic Laws: Nonlinear + large deformations
SED-stress relation, (Mooney-Rivlin, Neo-Hookean, ...)

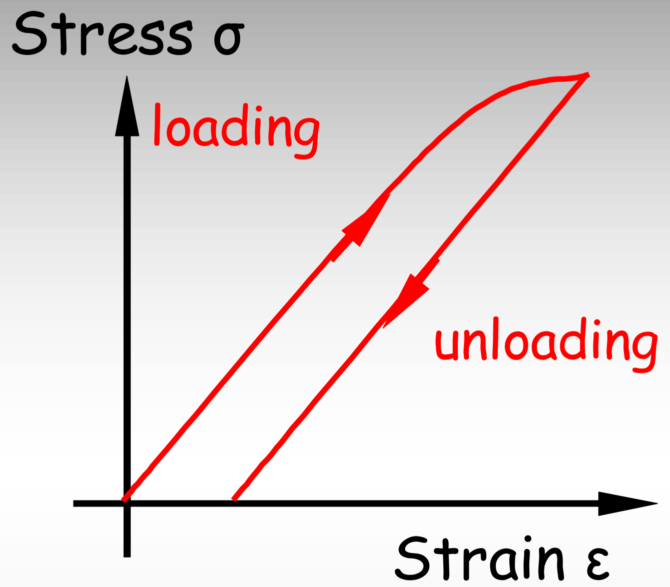
Properties for Ligaments

Properties	Value
Tensile strength	20 – 50 MPa
Shear strenght	1 kPa (fast loading 0.04mm/h) 740 kPa (slow loading 420mm/h)
Rupture strain	2 m/m (Shear) 0,1 – 0,5 m/m (Tension)
Rupture force	1500 N (Finger) 100 - 400 N (Ankle) 1200 N (ACL)
Stiffness	100 - 700 N/mm (ACL)
E Modulus, tension	100 – 1000 MPa

Non-elastic = plastic

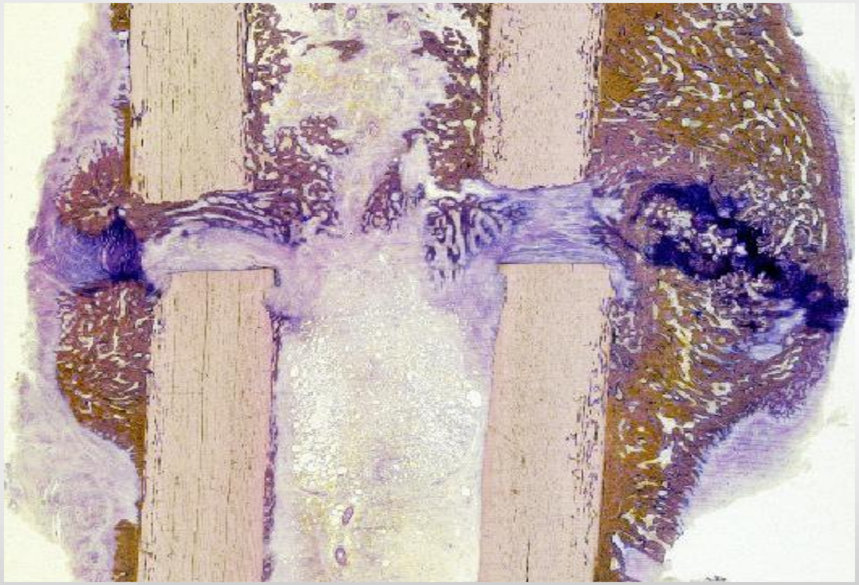
→ Bone strength

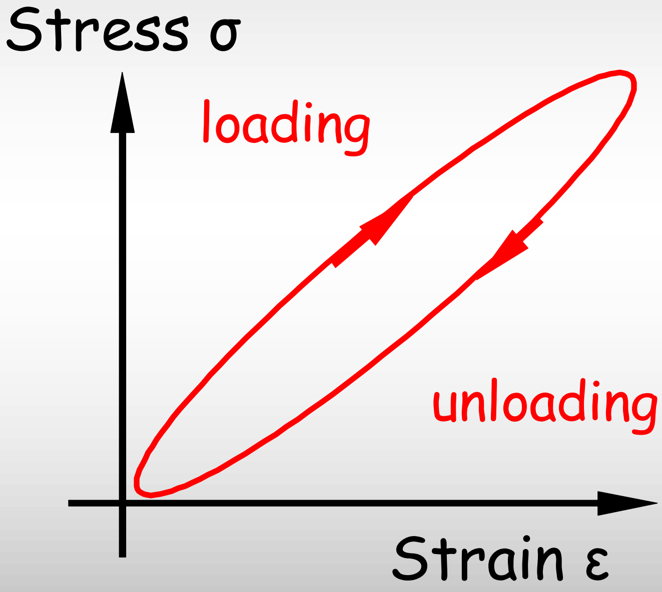




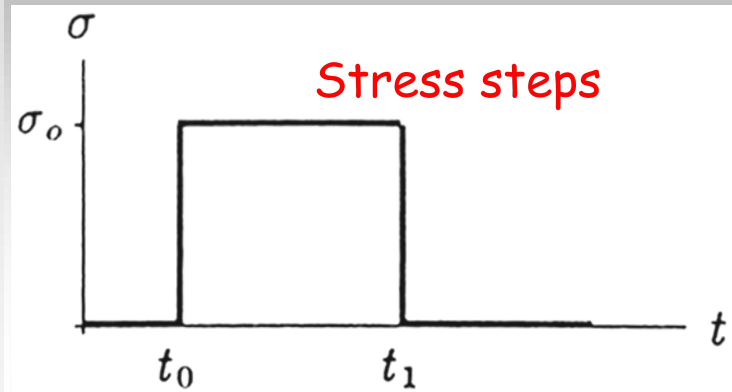
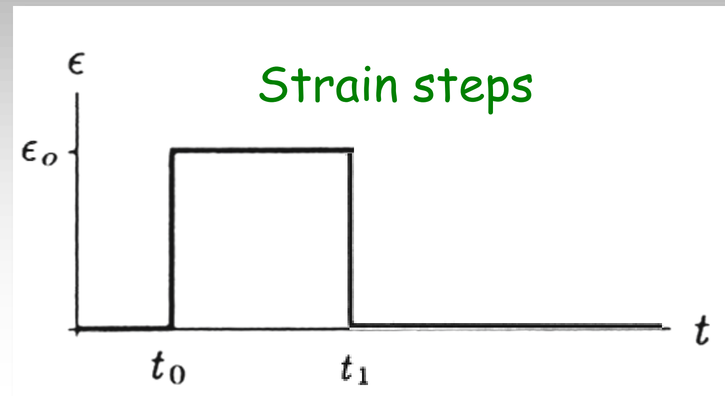
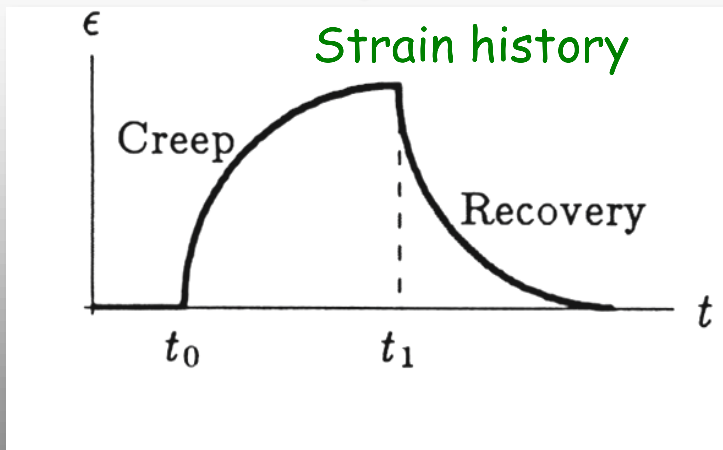
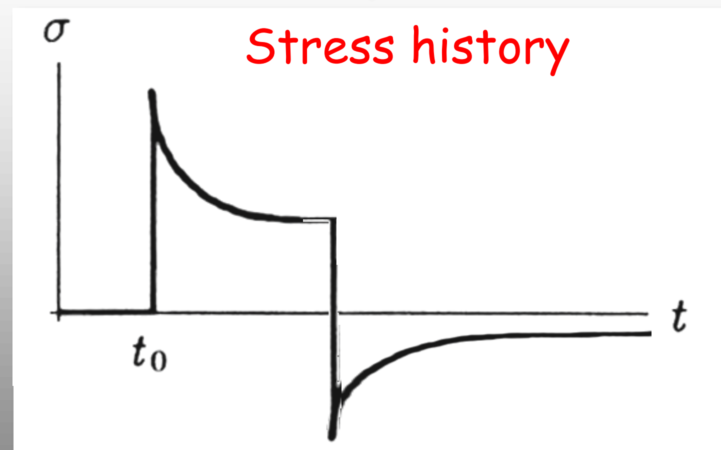
Viscoelastic Laws (Time Dependent)

→ Cartilage, Fibrous Tissue, Bone

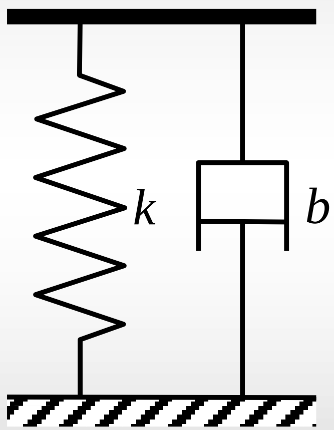




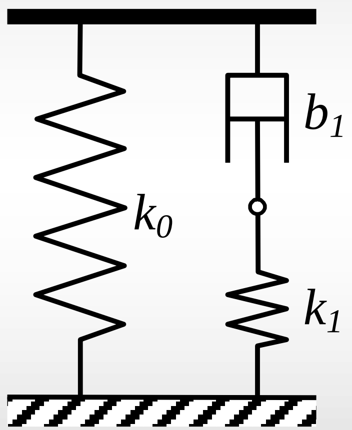
Effects of Viscoelastic Materials

	Retardation (Creep)	Relaxation
Applied		
Measured		

Viscoelastic Laws (Time Dependent)



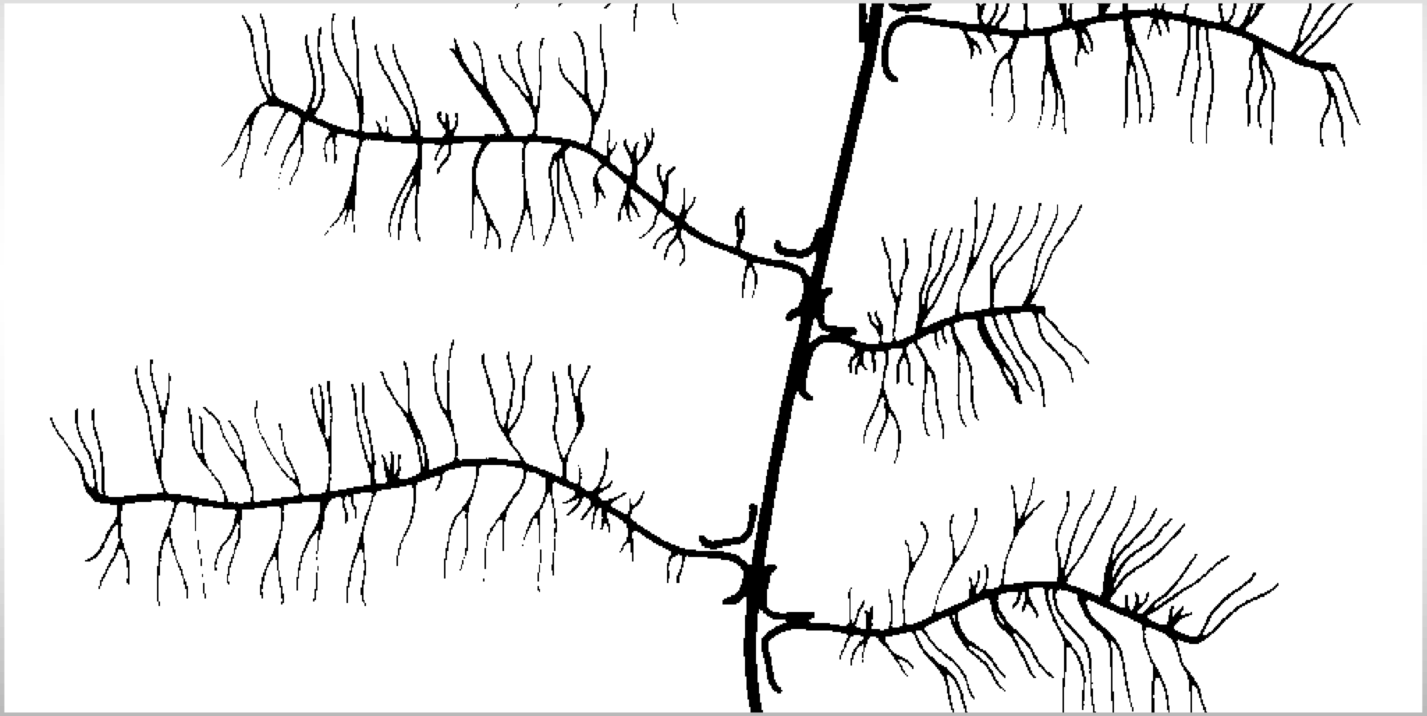
Typ:
Inner damping



Typ:
Memory effect

Poroelastic

→ Solid structure with pores: Cartilage, Fibrous Tissue, Bone



Hyaloron

Properties for Articular Cartilage

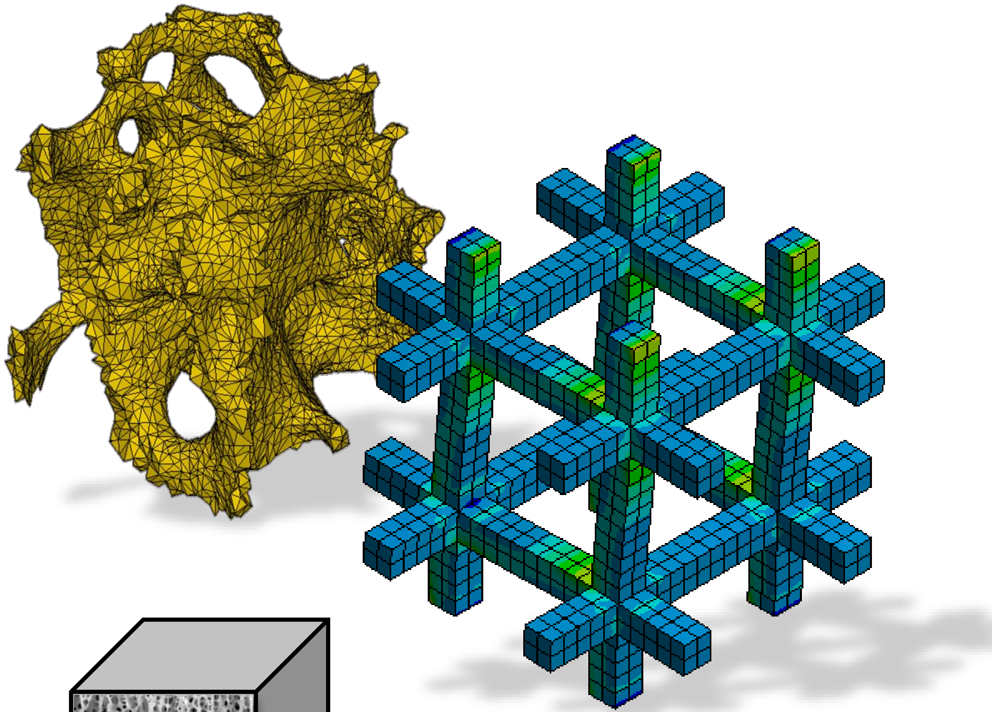
Properties	Values
Density	1300 kg/m^3
Water content	75%
Compressive strength	5 MPa
Tensile strenght	20 MPa
E Modulus, eq.	0,8 MPa
E Modulus, initial	3 MPa
Shear Modulus	3,5 MPa
Friction	0,005
Rupture strain	6...8%
Permeability	0,8 10^{-14} m^4/Ns

When is simplification allowed?

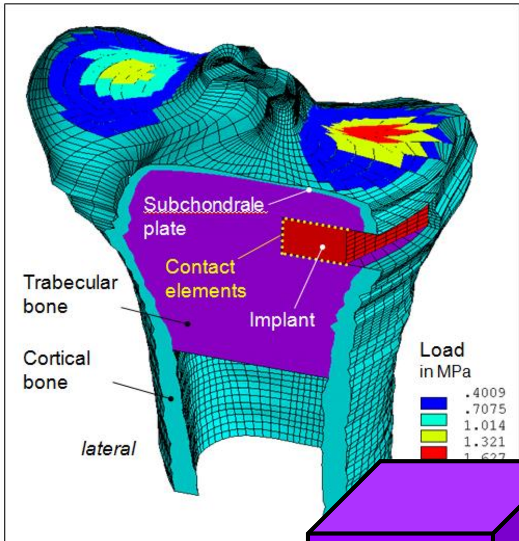
- Non linear: Small strains → „linear"
- Plastic: Approximation: „Ideal elastic plastic"
- Viscoelastic: Quasi-static Deformations → „elastic"
- Anisotropic: → „Worst-case scenario"

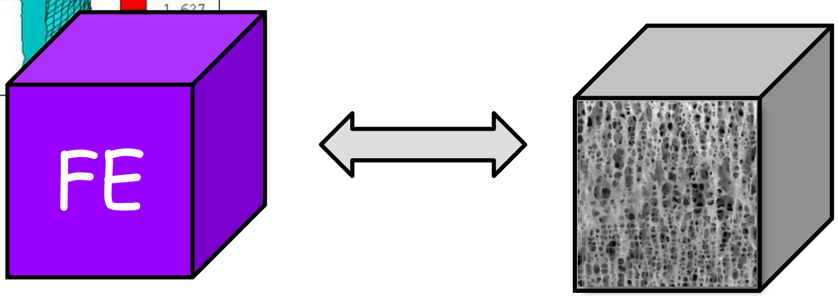
Modeling Trabecular Bone

Mikromodelle:



Makromodell:





$$E = 3790\,\dot{\epsilon}^{0.06}\,\rho^3 .$$

$$S = 68\,\dot{\epsilon}^{0.06}\,\rho^2$$

Carter & Hayes, 1977:
https://www.researchgate.net/publication/22580090_The_Compressive_Behavior_of_Bone_as_Two_Phase_Porous_Structure