Computational Biomechanics 2018

Lecture I:

Introduction, Basic Mechanics 1

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Scientific Computing Centre Ulm, UZWR
Ulm University

0 Organisation

Scientific Computing Centre Ulm
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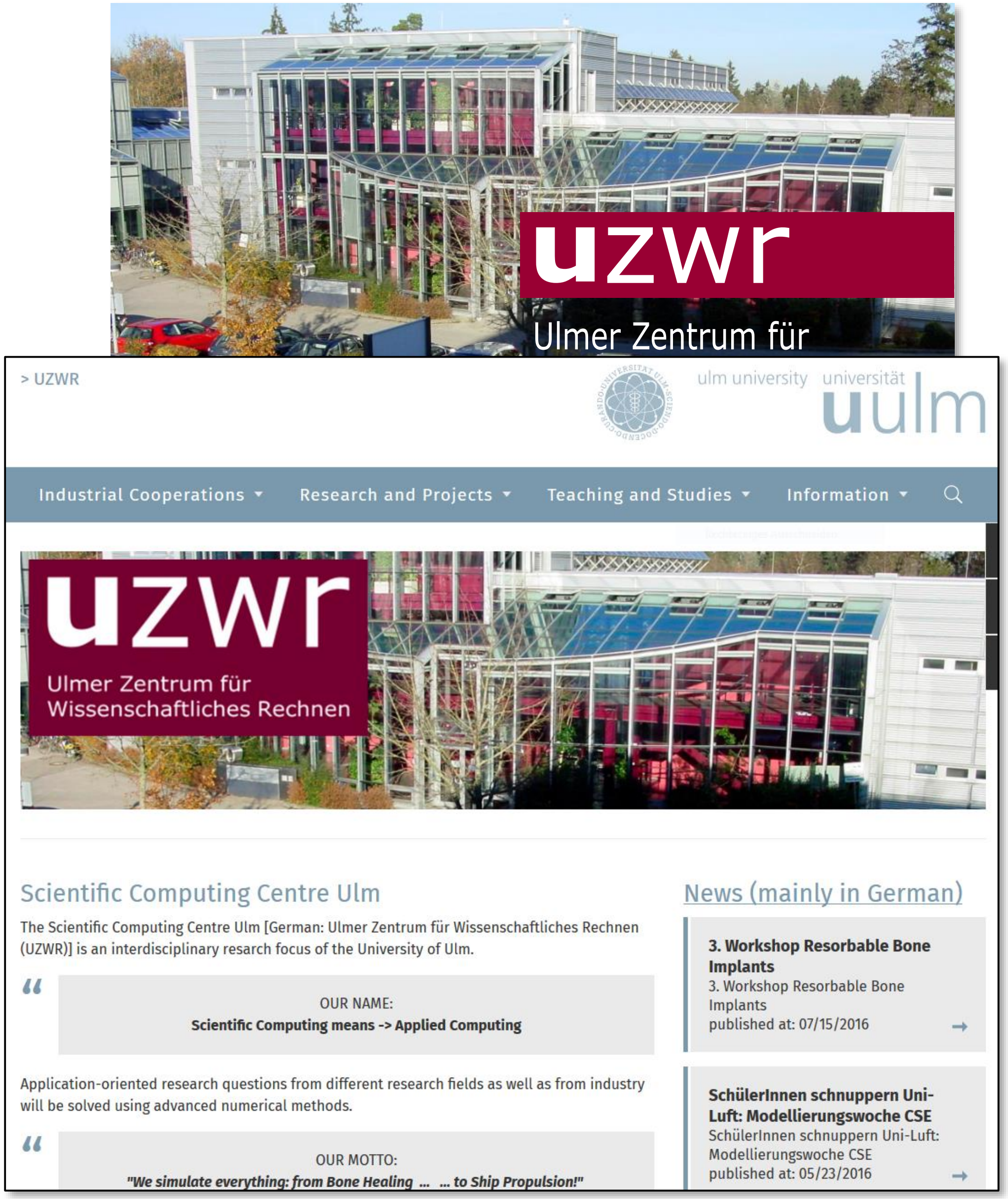
- English
- Rerun
- Times and Room
- Exam
- Max. 12 students
- Moodle
- Login to MAC

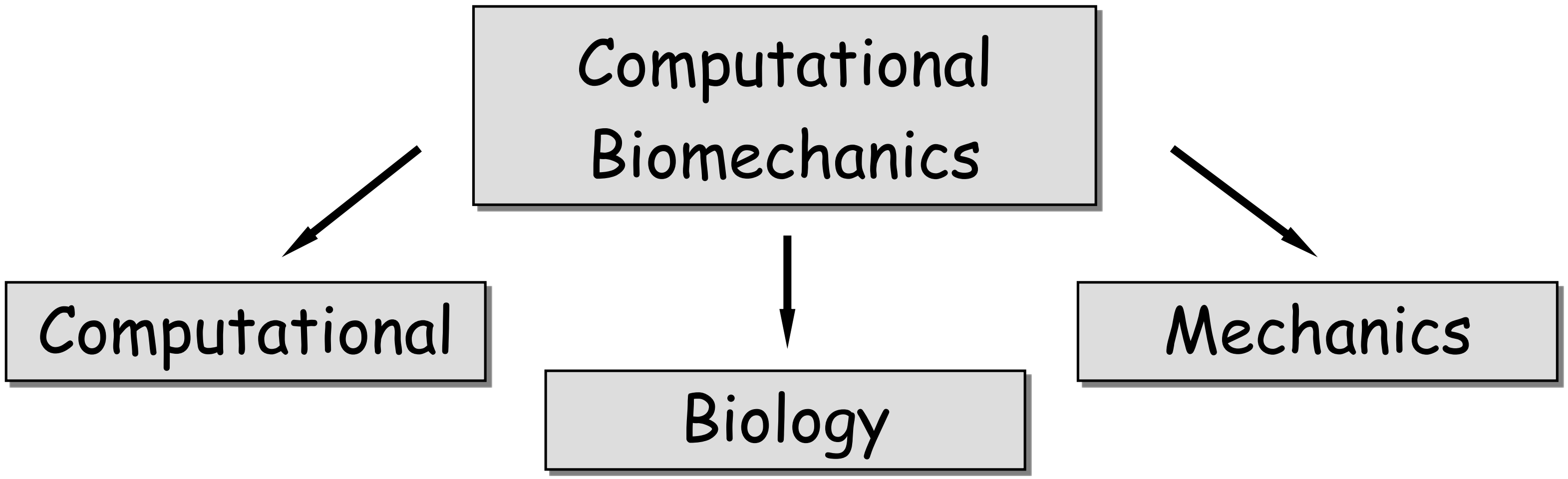


Contents

Detailed Schedule Summer 2018

No	Day	Date	Topics of Lecture and Laboratory	Lecturer
01	Mo	16 Apr	Lec: Intro to Biomechanics; Mechanical Basics 1 Lab: Intro to Ansys WB, Simple Bone Model	Ulli
02	Mo	23 Apr	Lec: Mechanical Basics 2 Lab: Loadcases, Stresses and Strains	Ulli
03	Mo	30 Apr	Lec: Material Properties of Biol. Tissues, Intro FEA Lab: Trabecular Bone Structural Model	Ulli
04	Mo	07 May	Lec: Forward Dynamics Lab: Forward Dyn., Multi Body Model with ADAMS	Lucas
05	Mo	14 May	Lec: Inverse Dynamics, Muscuoloskelettal Modells Lab: Inverse Dyn. Model with ANYBODY	Lucas
--	Mo	21 May	- Pentecost -	---
06	Mo	28 May	? Lab: KI, Medizin 4.0 ? ? Lec: Neuronales Netz ?	? Frank ?
07	Mo	04 Jun	Lec: From Clinical Imag Data to FE Model, Part 1 Lab: FE from CT Data	Matze
08	Mo	11 Jun	Lec: Bone Remodeling Lab: Remodeling of Trabecular Grid	Martin
09	Mo	18 Jun	Lec: Fracture Healing, Part 1 Lab: Implant Degradation and Bone Remodeling	Martin
10	Mo	25 Jun	Lec: Fracture Healing, Part 2 Lab: Healing Simulation Bone Chamber	Martin
11	Mo	02 Jul	Lec: Computational Fluid Dynamics Lab: Human Nose Air Flow Simulation	Lucas
12	Mo	09 Jul	Lec: From Clinical Imag Data to FE Model, Part 2 Lab: FE from CT Data 2	Matze
-	Mo	16 Jul	Oral Examinations A, 14:00, Office Simon, UZWR	All
-	Do	19 Jul	Oral Examinations B, 14:00, Office Simon, UZWR	All

1 General Information



<u>Biomechanics</u>: Solving biological questions using methods of mechanical engineering (Technische Mechanik), incl. experiments.

<u>Mechanobiology</u>: Reaction of biological structures on mechanical signals. Mechanotransduction: Molecular cell reactions.

Research Fields

Orthopaedic Biomechanics: Bone-implant contact, fracture healing, (artificial) joints, musculoskeletal systems, …

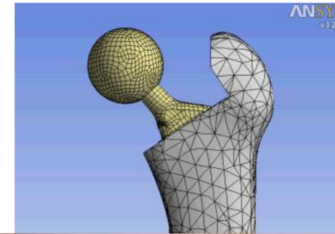
Dental Biomechanics: Dental implants, orthodontics, dental movements, brackets, …

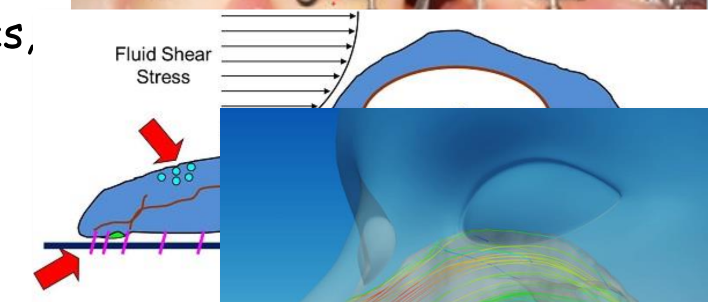
Cell Biomechanics: Cell experiments (cell gym) and simulations to study mechenotransduction

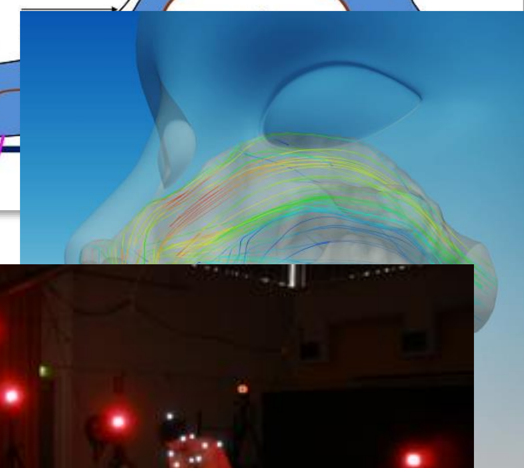
Fluid Biomechanics: Respiratory systems, blood flow, heart, …

Sport Biomechanics: Optimizing performance, techniques and equipment of competitive sports

Tree Biomechanics, Traffic Safety, Accident Research, …







Numerical Methods

Boundary Value Problems: Finite Elements: static structural analyses, displacements, stresses & strains, Finite Volumes: CFD

Initial Value Problems: Forward dynamics problems (biological and/or mechanical), multi-body systems, musculoskeletal systems, movements, inverse dynamics problem: calculating muscle forces from measured movements

Multiscale Modeling: To handle higly complex systems

Model Reduction: dito

Fuzzy Logic: Fracture healing in Ulm

...

Mechanical Basics

1.3 Variables, Dimensions and Units

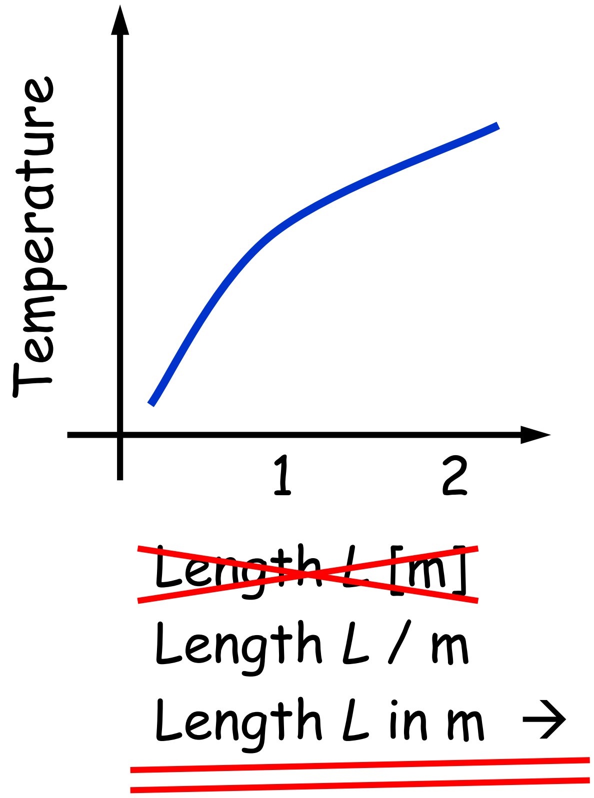
Standard: ISO 31, DIN 1313

Variable = Number · Unit

Length L = $2 \cdot$ m = 2 m

{Variable} = Number

[Variable] = Unit



Three mechanical SI-Units:

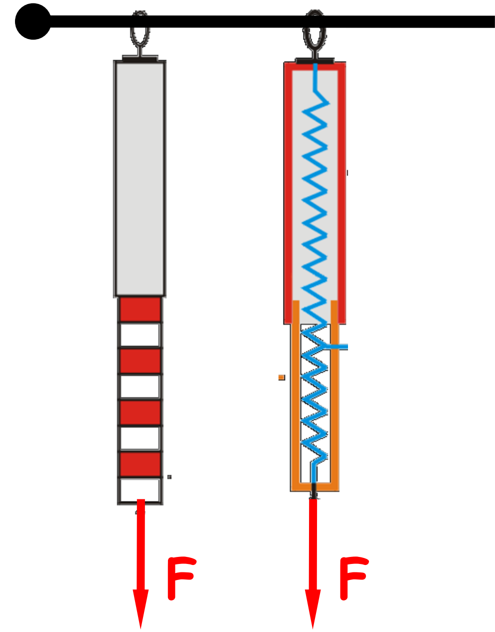
m (Meter)

kg (Kilogram)

s (Seconds)

2 STATICS OF RIGID BODIES

2.1 Force



- We all believe to know what a *force* is.
- But, *force* is an invention not a discovery!
- … it can not be measured directly.

Newton's 2nd Law [Axiom]:

Force = Mass times Acceleration or $F = m \cdot a$

Note to Remember:

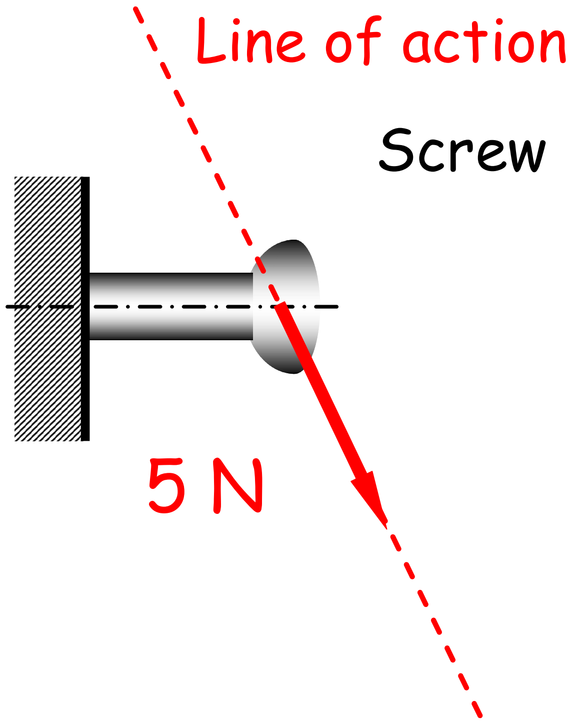
„A force is the cause of acceleration or deformation of a body"

Representation of Forces

... with arrows

Forces are Vectors with

- **Magnitude**
- **Direction**
- **Sense of Direction**



Units of Force

Newton

N = kg·m/s²

$F_G = m \cdot g = 0{,}1\ \text{kg} \cdot 9{,}81\ \text{m/s}^2$

$= 0{,}981\ \text{kg m/s}^2$

$\approx 1\ \text{N}$

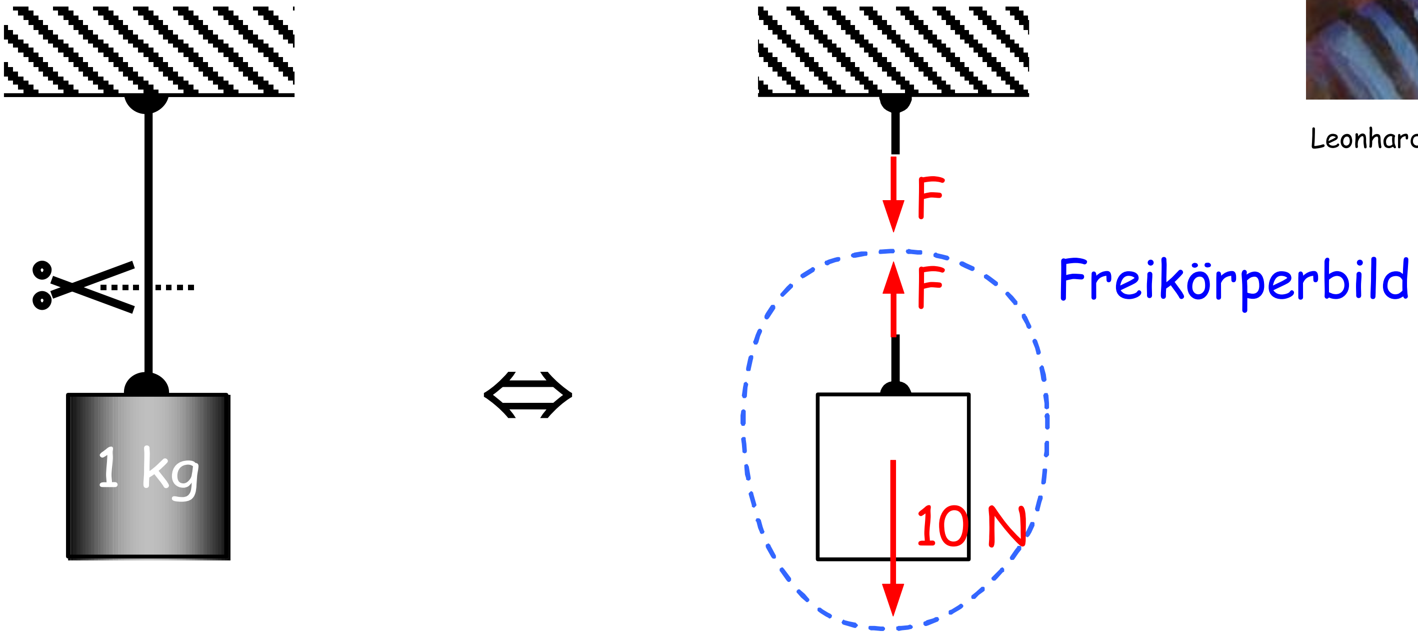
Note to Remember:

1 Newton ≈ Weight of a bar of chocolate (100 g)

2.2 Method of Sections (Euler) [Schnittprinzip]

Leonhard Euler, 1707 - 1783

Free-Body Diagramm (FBD) [Freikörper-Bild]

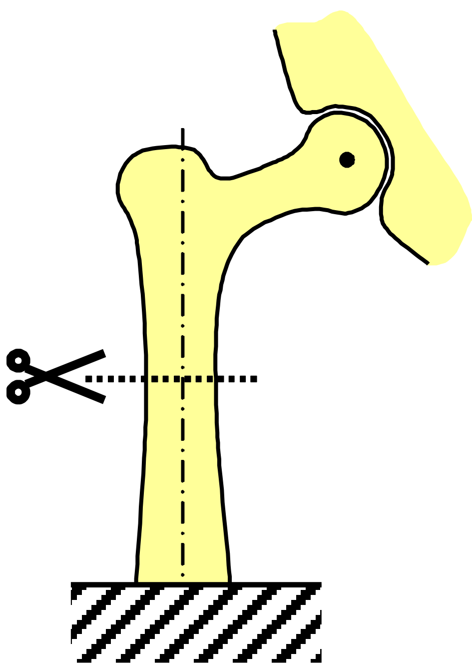


Note to Remember:

First, cut the system, then include forces and moments.

Free-body diagram = <u>completely</u> isolated part.

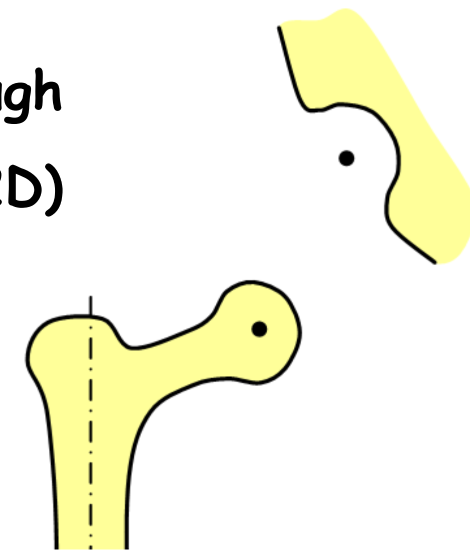
2.2 Method of Sections



⇔

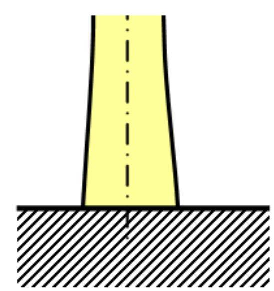
Cut through

joint (2D)

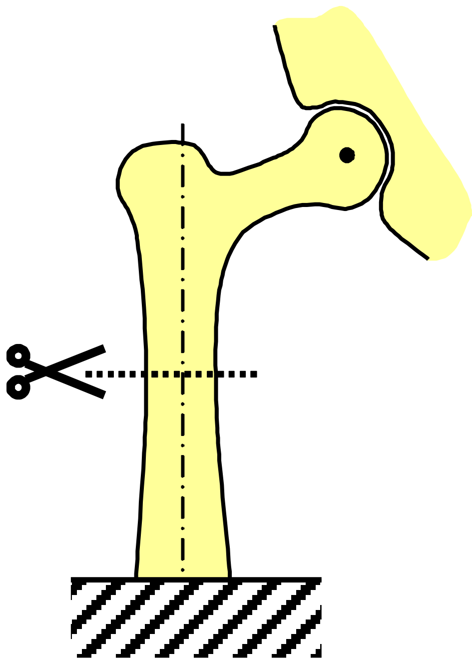


Cut through

bone (2D)

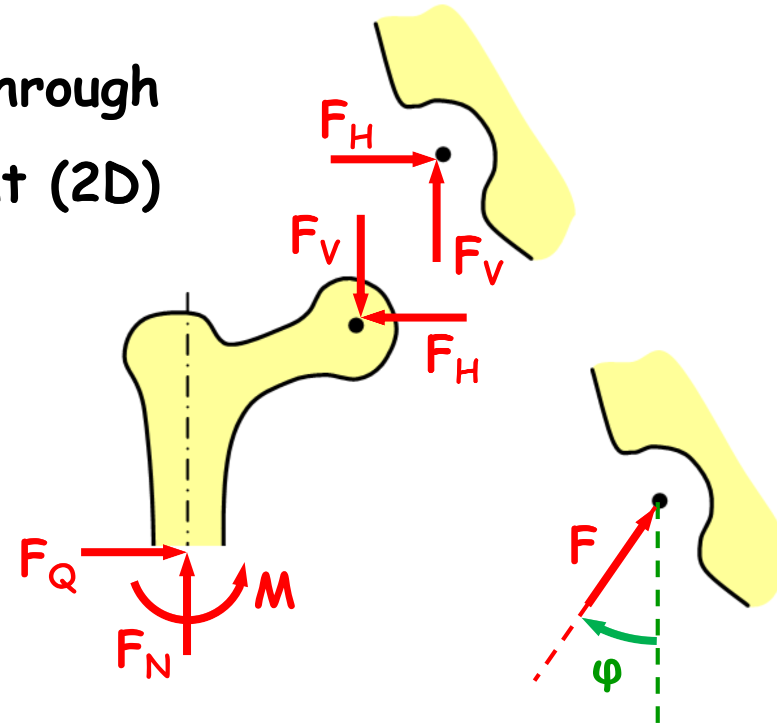


2.2 Method of Sections

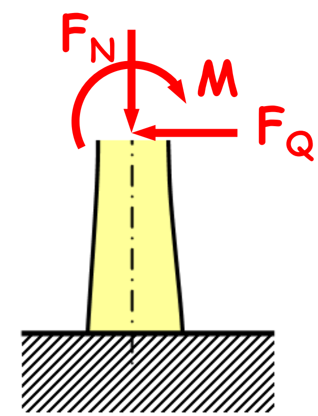


⇔

Cut through joint (2D)

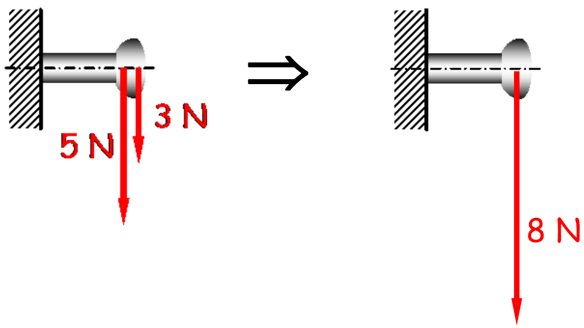


Cut through bone (2D)

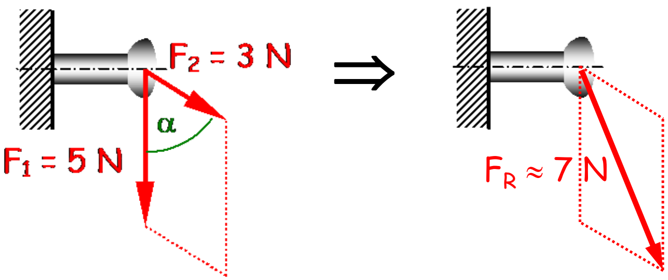


2.3 Combining and Decomposing Forces

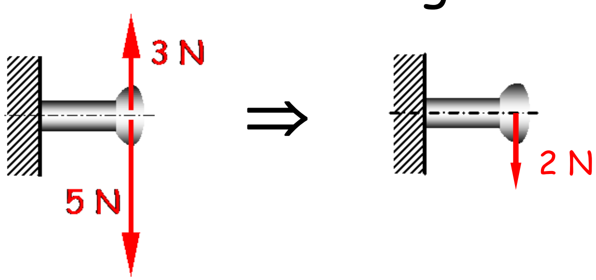
Summation of Magnitudes



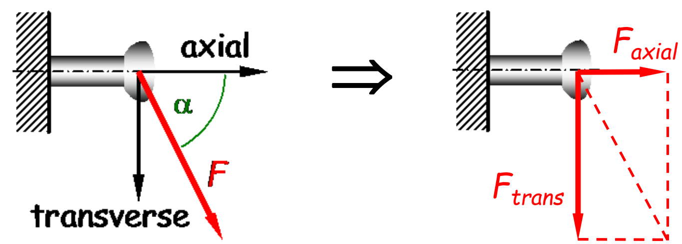
Vector Addition



Subtraction of Magnitudes

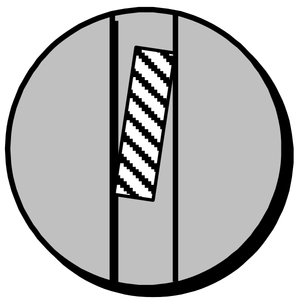


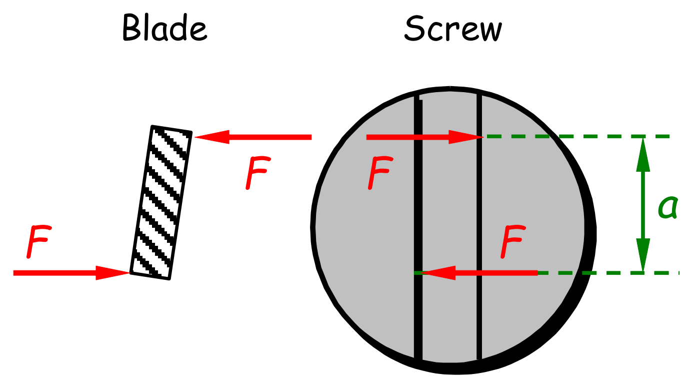
Decomposition into Components



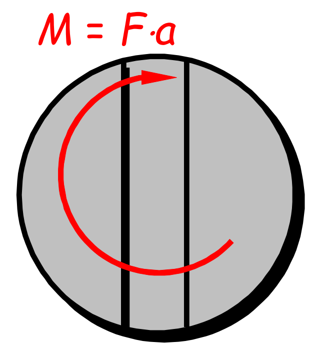
2.4 The Moment [Das Moment]

Slotted screw with screwdriver blade





Force Couples (F, a)



Moment M

Note to remember:

The moment $M = F \cdot a$ is equivalent to a force couple (F, a).

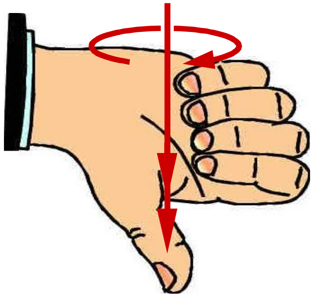
A moment is the cause for angular acceleration or angular deformation (Torsion, Bending) of a body.

Units for Moment

Newton-Meter

$N \cdot m = kg \cdot m^2/s^2$

Rechte-Hand-Regel:

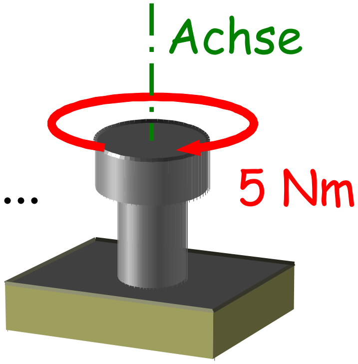


Representation of Moments

... with rotation arrows or double arrows

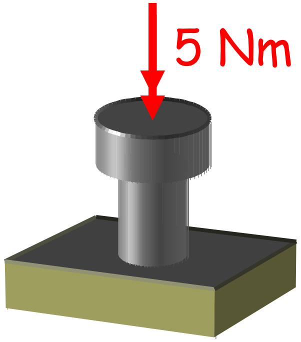
Moments are Vectors with ...

- **Magnitude**
- **Direction**
- **Sense of Direction**



Drehpfeil

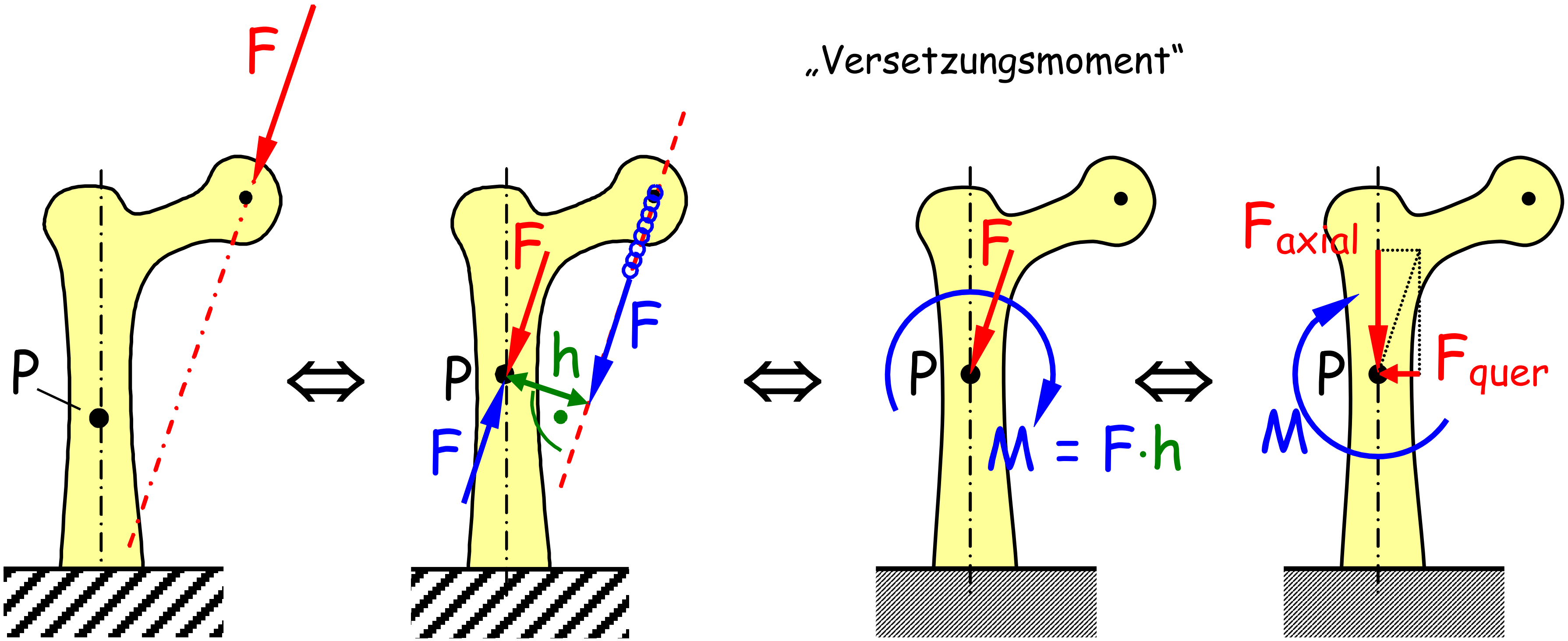
oder



Doppelpfeil

2.5 Moment of a Force about a Point [Versetzungsmoment]

„Versetzungsmoment“



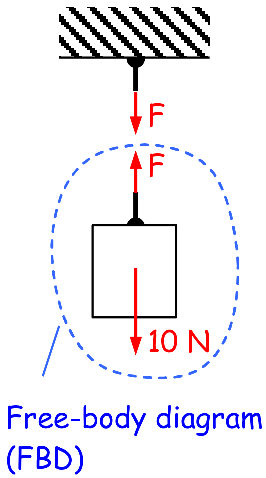
Note to Remember:

Moment = Force times lever-arm

2.7 Static Equilibrium

Important:

Free-body diagram (FBD) first, then equilibrium!

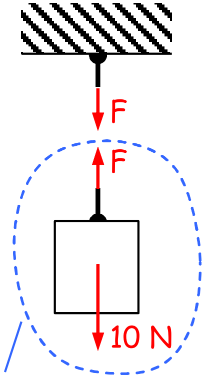


For 2D Problems max. 3 equations for each FBD:

The sum of all forces in *x*-direction equals zero:	$F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0$
The sum of all forces in *y*-direction equals zero:	$F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0$
The sum of Moments with respect to P equals zero:	$M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0$

(For 3D Problems max. 6 equations for each FBD)

2.7 Static Equilibrium



Free-body diagram (FBD)

Important:

Free-body diagram (FBD) first, then equilibrium!

3 equations of equilibrium for each FBD in 2D:

$$\text{Sum of all forces in x-direction:} \quad F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0,$$

$$\text{Sum of all forces in y-direction:} \quad F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0,$$

$$\text{Sum of all moments w.resp. to P:} \quad M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0.$$

- Force EEs can be substituted by moment EEs
- 3 moment reference points should not lie on one line

6 equlibrium equations for one FBD in 3D:

Summe aller Kräfte in x-Richtung: $\sum_i F_{ix} \overset{!}{=} 0,$

Summe aller Kräfte in y-Richtung: $\sum_i F_{iy} \overset{!}{=} 0,$

Summe aller Kräfte in z-Richtung: $\sum_i F_{iz} \overset{!}{=} 0,$

Summe aller Momente um x-Achse bezüglich Punkt P: $\sum_i M_{ix}^{P} \overset{!}{=} 0.$

Summe aller Momente um y-Achse bezüglich Punkt Q: $\sum_i M_{iy}^{Q} \overset{!}{=} 0.$

Summe aller Momente um z-Achse bezüglich Punkt R: $\sum_i M_{iz}^{R} \overset{!}{=} 0.$

- Force EEs can be substituted by moment EEs
- Max. 2 moment axis parallel to each other
- Determinant of coef. matrix not zero

2.8 Recipe for Solving Problems in Statics

Step 1: Model building. Generate a simplified replacement model (diagram with geometry, forces, constraints).

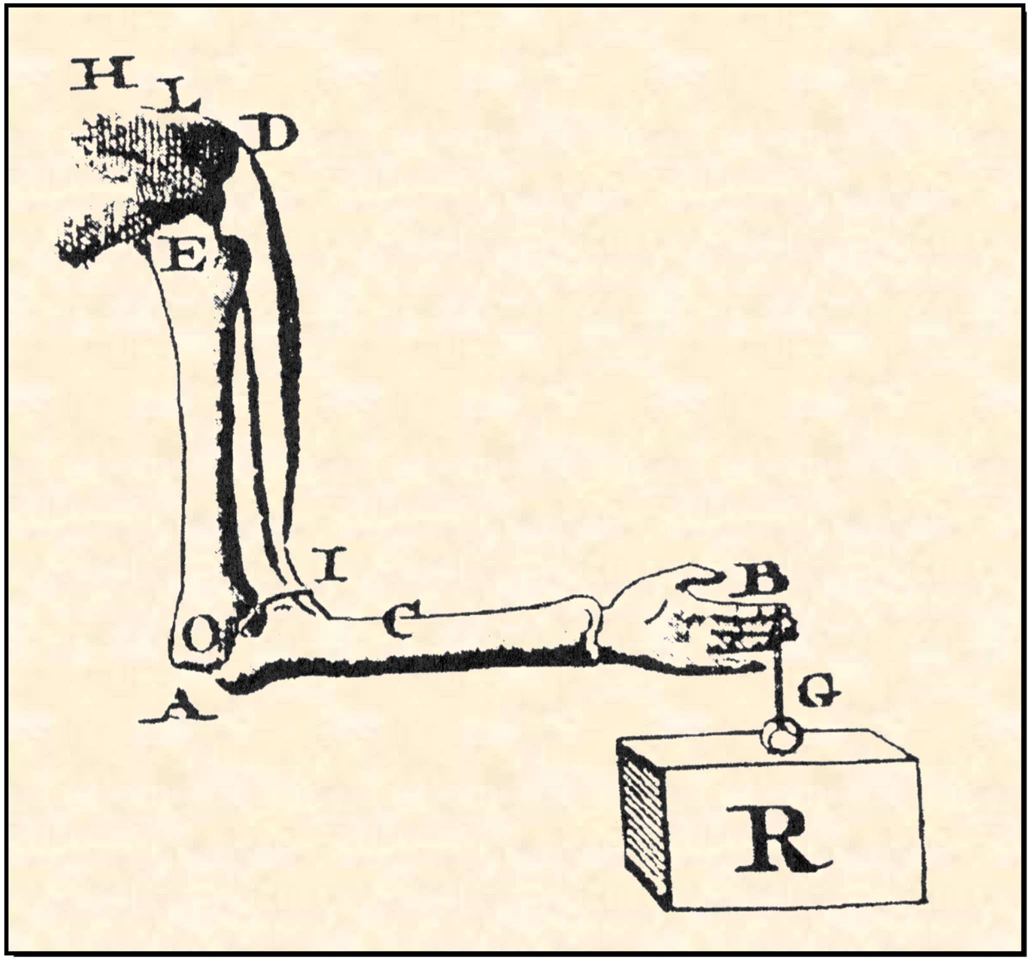
Step 2: Cutting, Free-body diagram. Cut system and develop free-body diagrams. Include forces and moments at cut, as well as weight.

Step 3: Equilibrium equations. Write the force- and moment equilibrium equations (only for free-body diagrams).

Step 4: Solve the equations. One can only solve for as many unknowns as equations, at most.

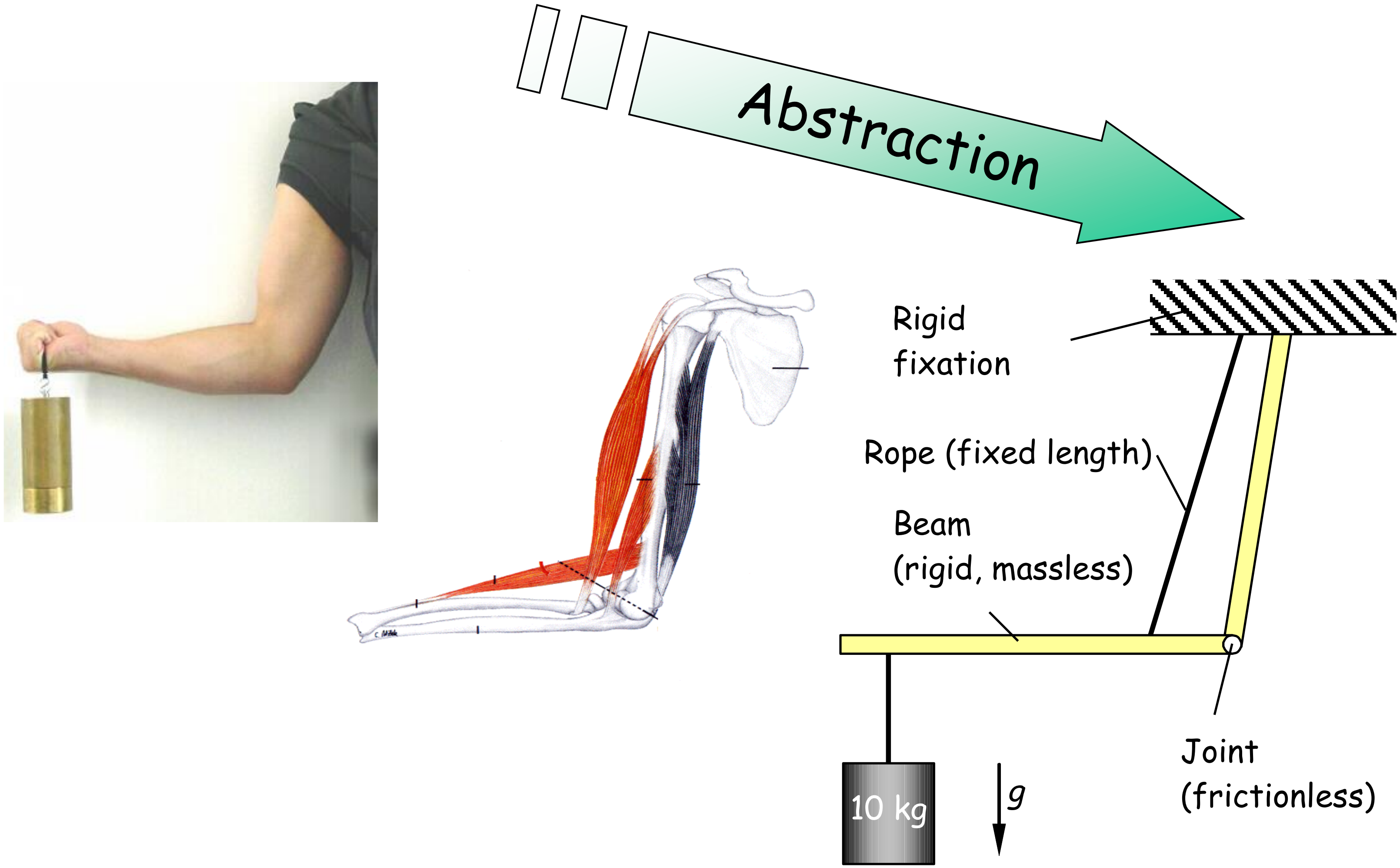
Step 5: Display results, explain, confirm with experimental comparisons. Are the results reasonable?

2.9 Classical Example: „Biceps Force“

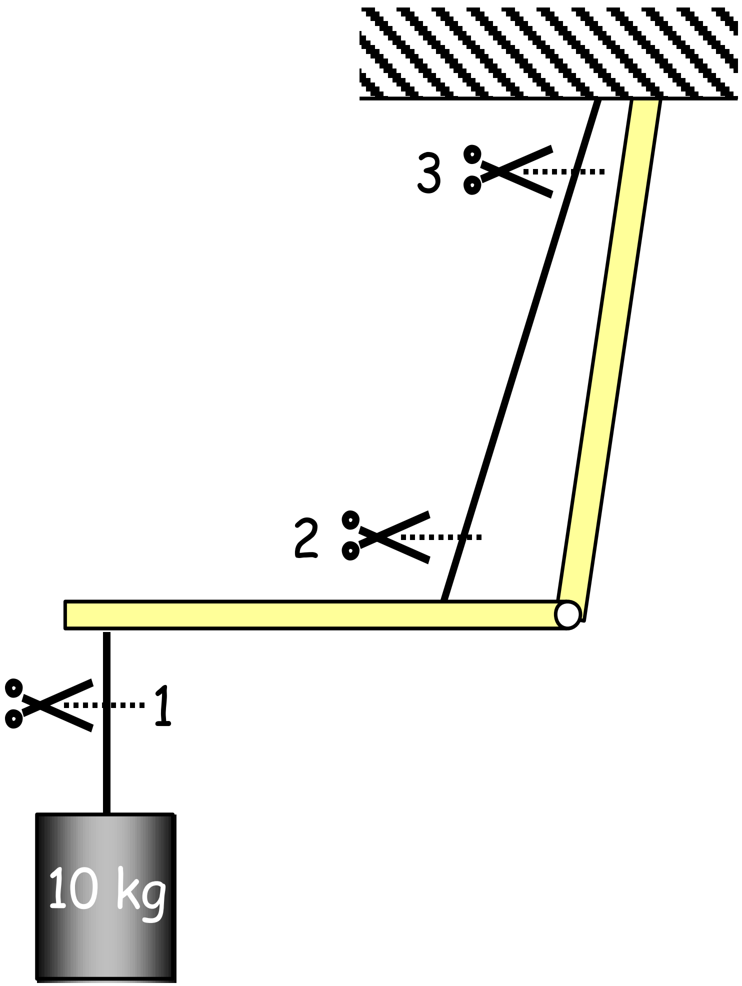


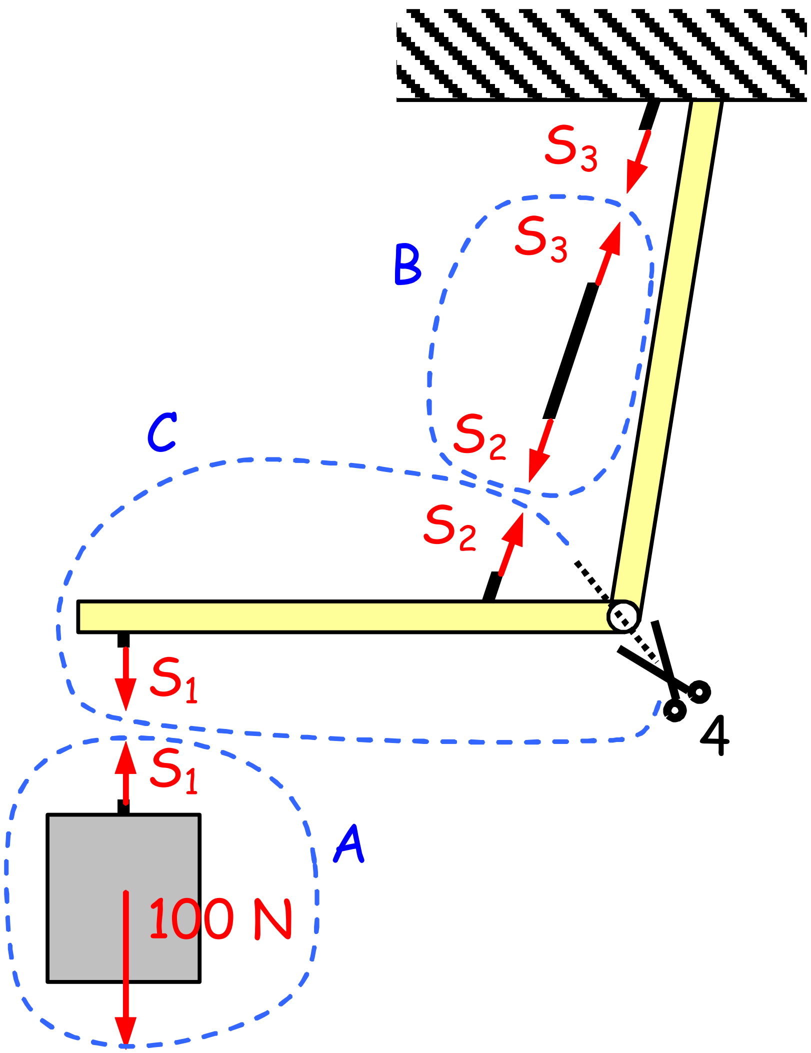
From:
„De Motu Animalium“
G.A. BORELLI
(1608-1679)

Step 1: Model building

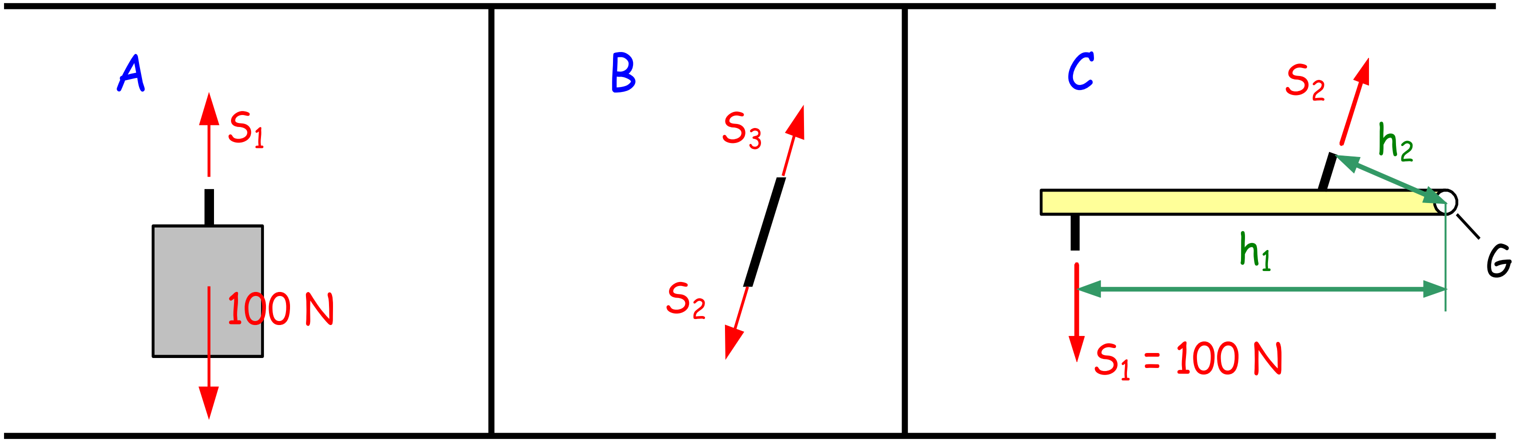


Schritt 2: Schneiden und Freikörperbilder





More to Step 2: Cutting and Free-Body Diagrams



Step 3 and 4: Equilibrium and Solving the Equations

Sum of all forces in vertical direction = 0	Sum of all forces in "rope" direction = 0	Sum of all moments with respect to Point G = 0
$100\,\mathrm{N} + (-S_1) \overset{!}{=} 0$ $\Rightarrow \underline{\underline{S_1 = 100\,\mathrm{N}}}$	$S_2 + (-S_3) \overset{!}{=} 0$ $\Rightarrow S_3 = S_2$	$-S_1 \cdot h_1 + S_2 \cdot h_2 \overset{!}{=} 0$ $-100\,\mathrm{N} \cdot 35\,\mathrm{cm} + S_2 \cdot 5\,\mathrm{cm} \overset{!}{=} 0$ $\Rightarrow S_2 = 100\,\mathrm{N} \cdot \frac{35\,\mathrm{cm}}{5\,\mathrm{cm}} = \underline{\underline{700\,\mathrm{N}}}$

3.1 Stresses

... to account for the loading of the material !

Fotos: Lutz Dürselen

Note to Remember:

Stress = „smeared“ force

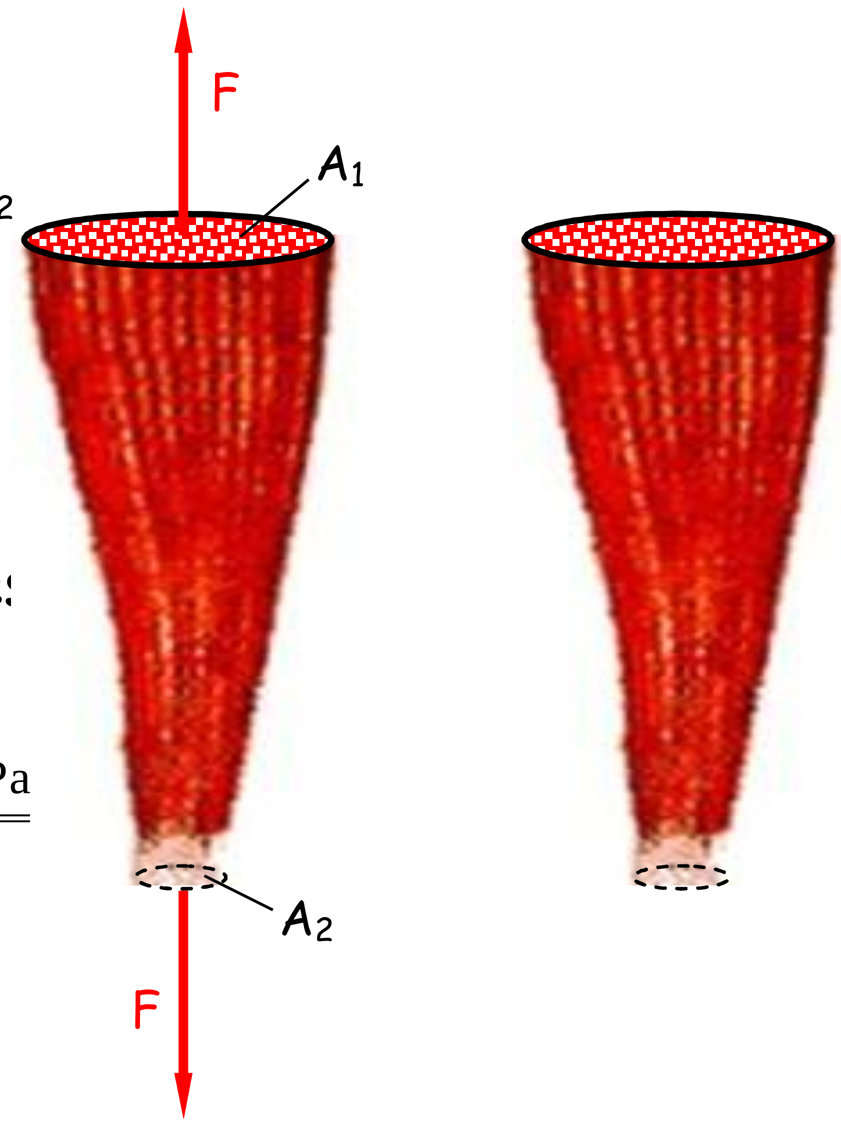
Stress = Force per Area or $\sigma = F/A$

(Analogy: „Nutella bread teast “)

Units of Stress

Pascal: $1\,Pa = 1\,N/m^2$

Mega-Pascal: $1\,MPa = 1\,N/mm^2$



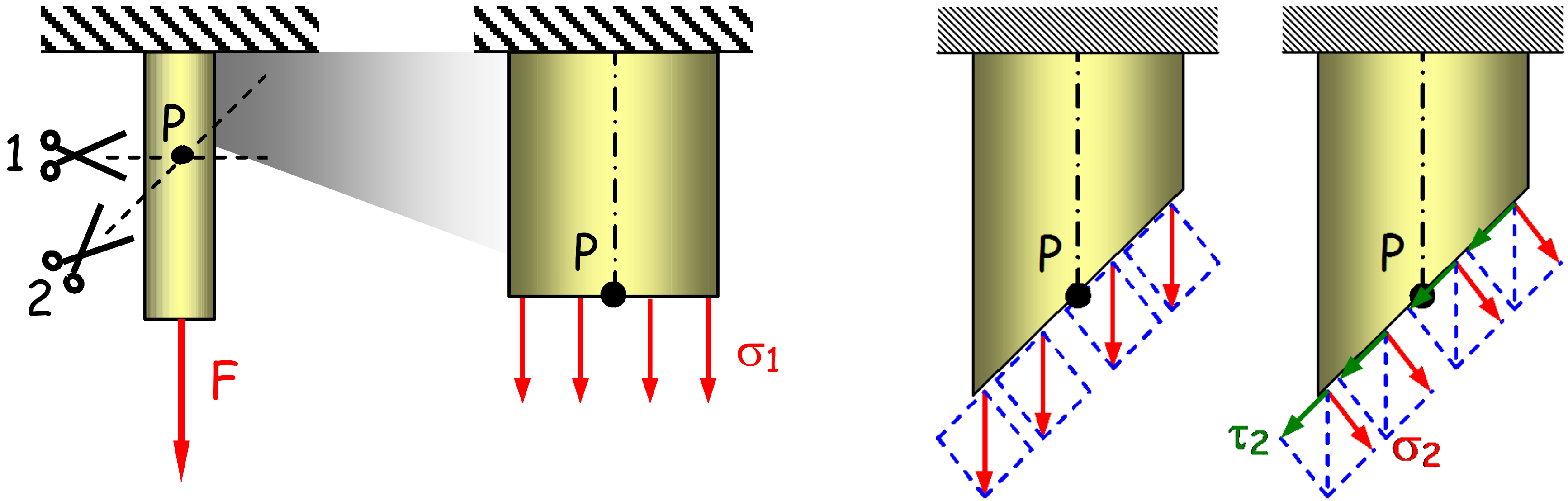
3.2 Example:

"Tensile stress in Muscle:

$$\sigma_1 = \frac{F}{A_1} = \frac{700\,N}{7000\,mm^2} = 0{,}1\frac{N}{mm^2} = \underline{\underline{0{,}1\,MPa}}$$

$$\sigma_2 = \frac{F}{A_2} = \frac{700\,N}{70\,mm^2} = 10\frac{N}{mm^2} = \underline{\underline{10\,MPa}}$$

3.3 Normal and Shear Stresses



Tensile bar

Cut 1:
Normal stress σ_1

Cut 2:
Normal stress σ_2
Shear stress τ_2

Note to Remember:

First, you must choose a point and a **cut** through the point, **then** you can specify (type of) **stresses** at this point in the body.

Normal stresses (tensile and compressive stress) are oriented perpendicular to the cut-surface.

Shear stresses lie tangential to the cut-surface.

General (3D) Stress State: Stress Tensor

... in one point of the body: **How much numbers do we need?**

- 3 stress components in one cut (normal str., 2x shear str.)

  times

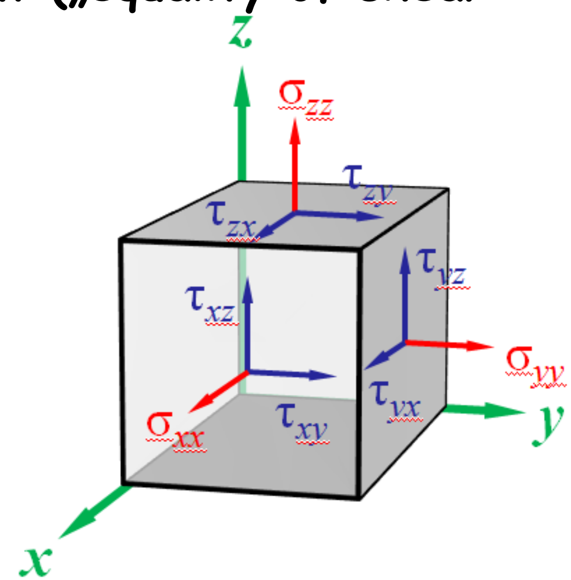
- 3 cuts (e.g. frontal, sagittal, transversal)

  results in

- 9 stress components for the full stress state in the point.
- But only 6 components are linear independent („equality of shear stresses“)

„Stress Tensor“

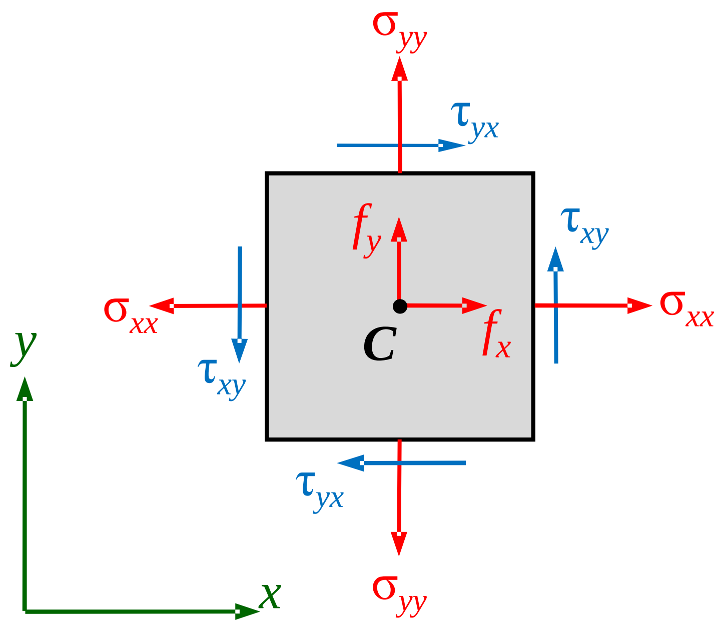
$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$



Symmetry of the Stress Tensor

Boltzmann Continua: Only volume forces (f_x und f_y), no volume moments assumed

→ „Equality of corresponding shear stresses"

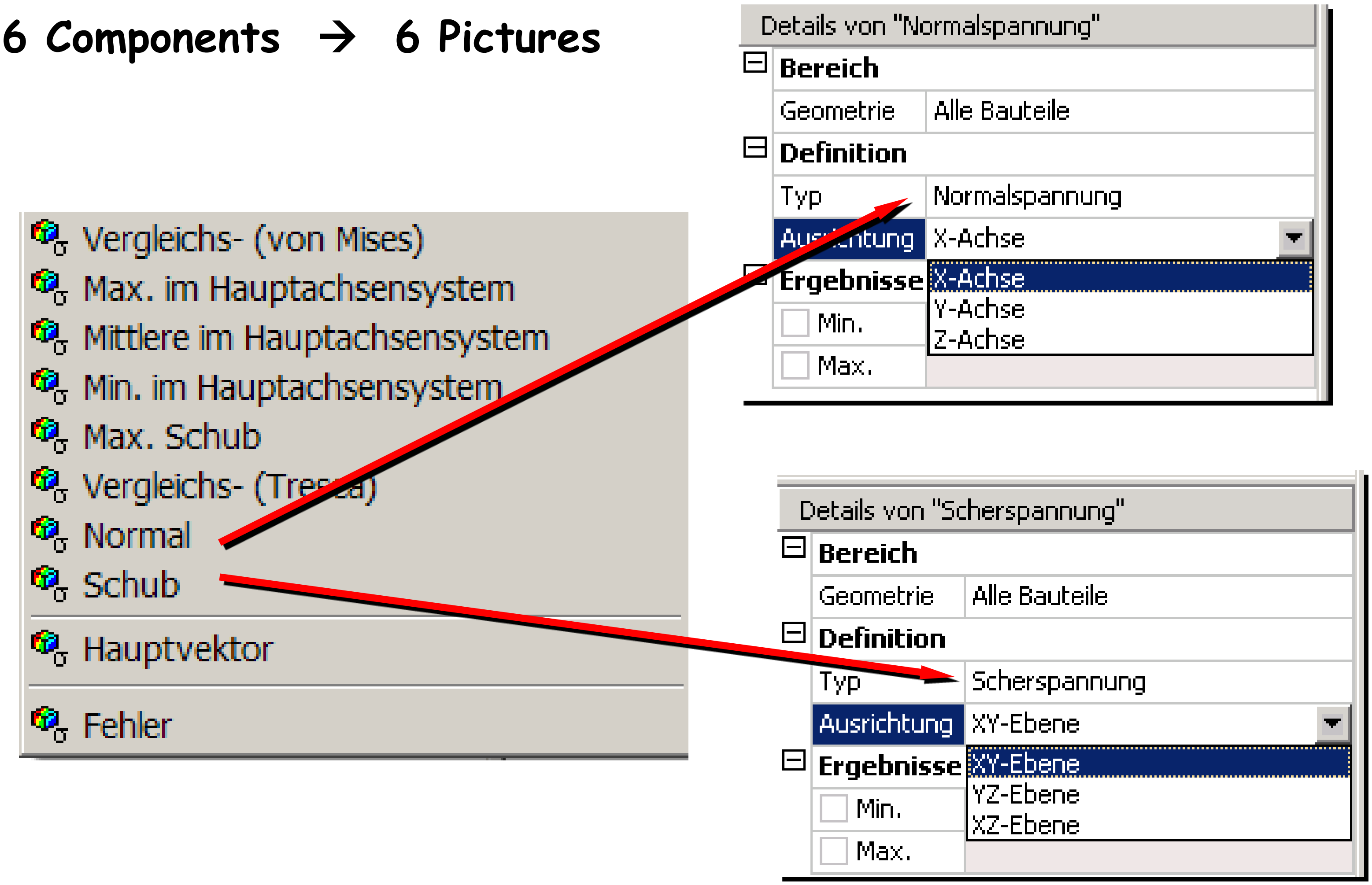


$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ . & \sigma_{yy} & \tau_{yz} \\ \text{sym} & . & \sigma_{zz} \end{bmatrix}$$

$$\sum M^{(C)} = 2 \cdot \underbrace{\tau_{xy} \Delta y \Delta z}_{\text{Kraft}} \cdot \underbrace{\frac{1}{2} \Delta x}_{\text{Hebelarm}} - 2 \cdot \underbrace{\tau_{yx} \Delta x \Delta z}_{\text{Kraft}} \cdot \underbrace{\frac{1}{2} \Delta y}_{\text{Hebelarm}} = 0.$$

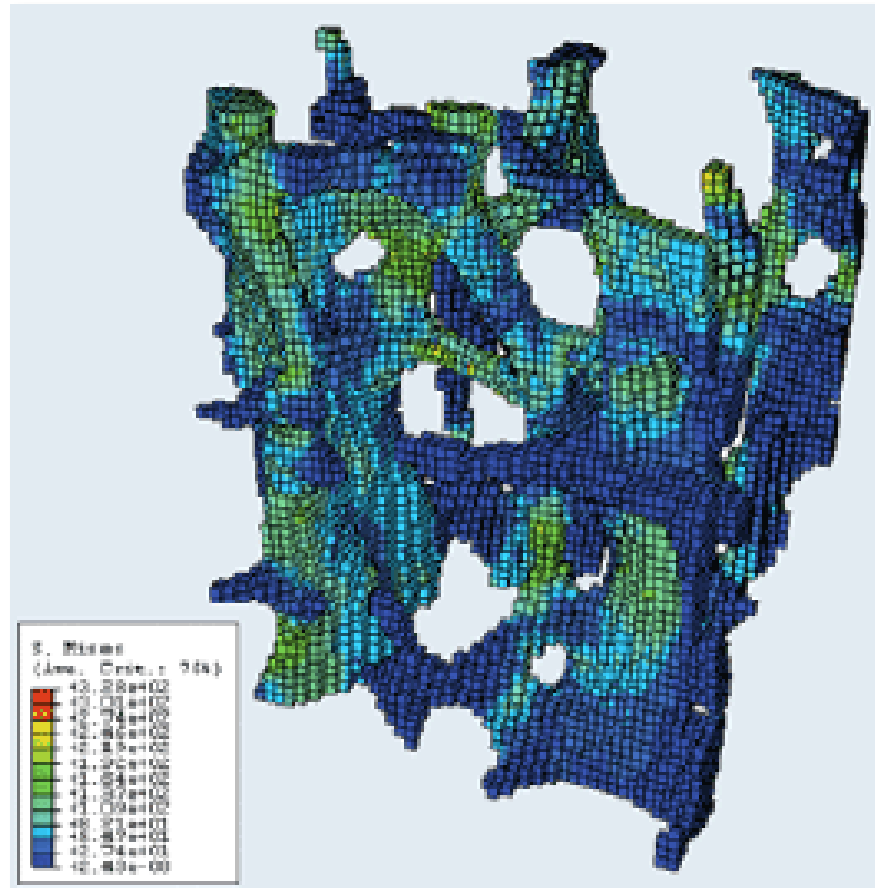
General 3D Stress State

6 Components → 6 Pictures



Problem:

- How to produce nice Pictures?
- Which component should I use?
- Do I need 6 pictures at the same time?



So called „Invariants“ are „smart mixtures“ of the components

- Vergleichs- (von Mises)
- Max. im Hauptachsensystem
- Mittlere im Hauptachsensystem
- Min. im Hauptachsensystem
- Max. Schub
- Vergleichs- (Tresca)
- Normal
- Schub

$$\sigma_{Mises} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\,\sigma_{yy} - \sigma_{xx}\,\sigma_{zz} - \sigma_{yy}\,\sigma_{zz} + 3\,\tau_{xy}^2 + 3\,\tau_{xz}^2 + 3\,\tau_{yz}^2}$$

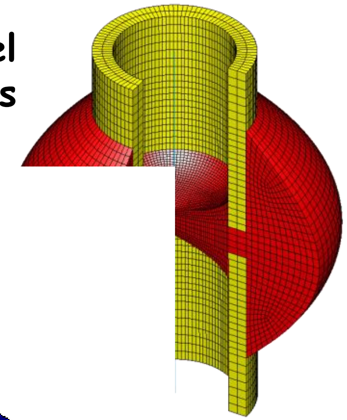
3.4 Strains

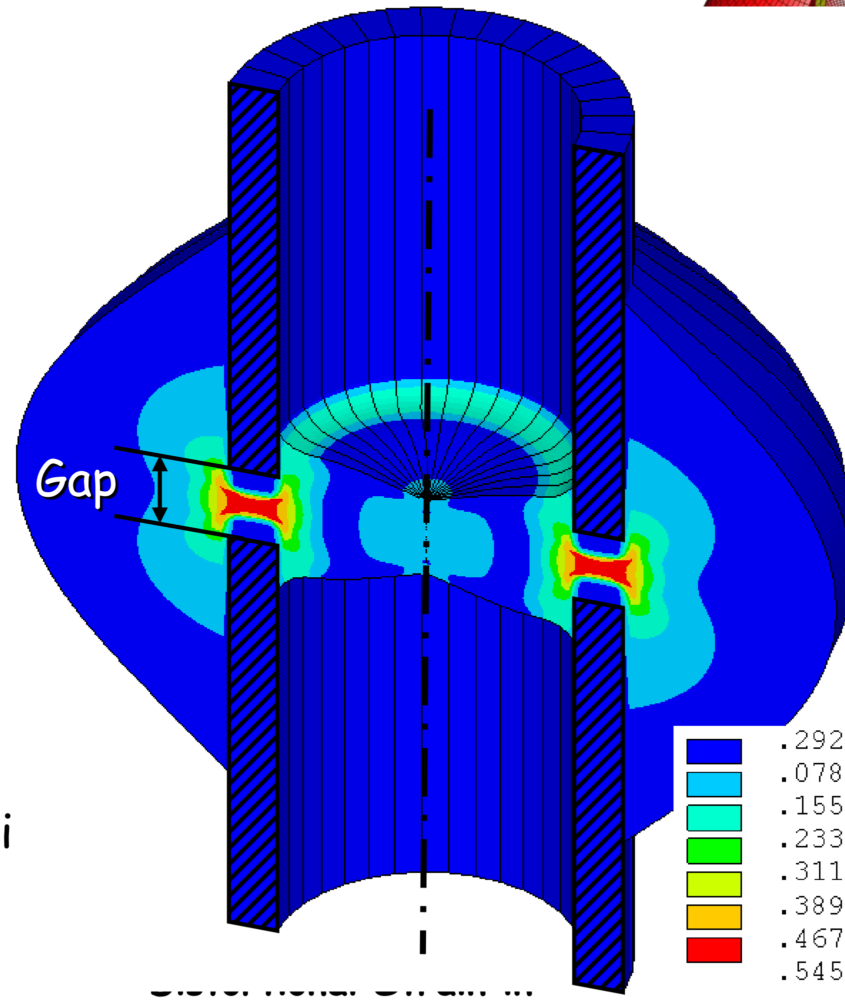
- Global, (external) strains

$$\varepsilon := \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L_0}$$

- Local, (internal) strains

Finite element model
of the fracture callus





a fracture callus

Units of Strain

without a unit

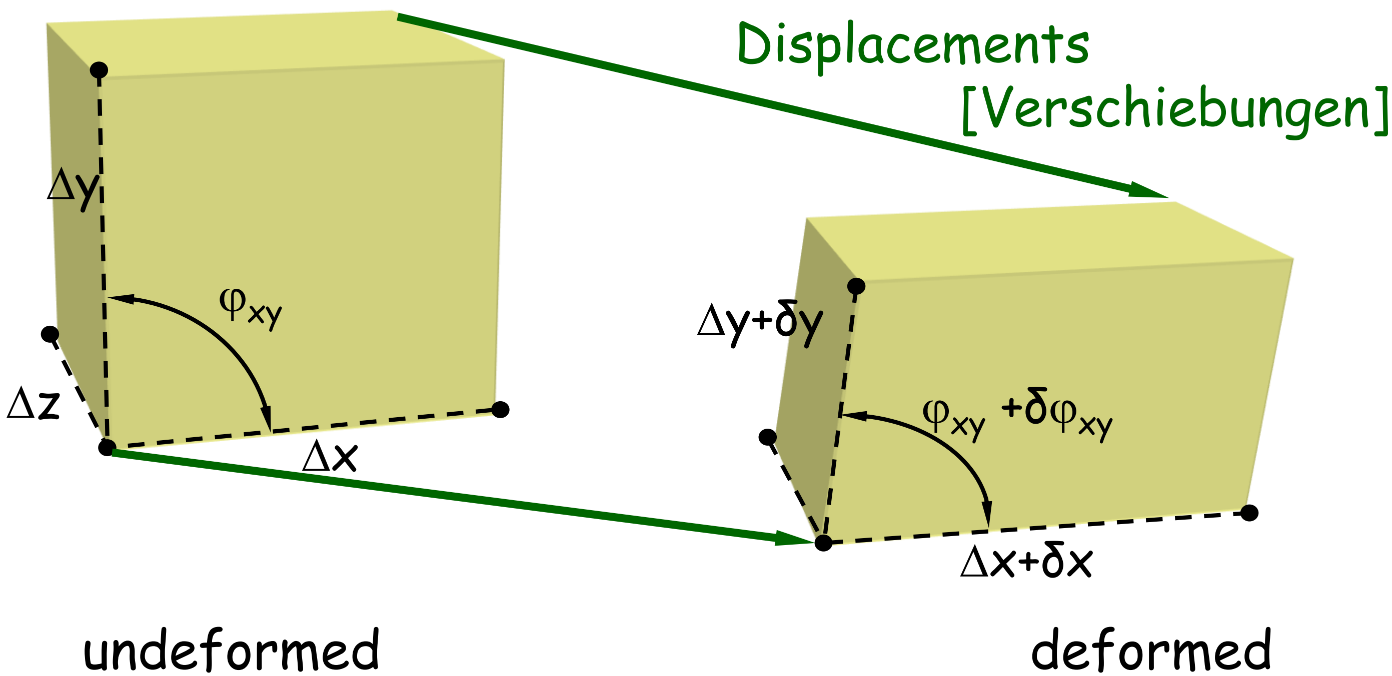
1

1/100 = %

1/1.000.000 = με (micro strai

= 0,1 %

3D Local Strain State: Strain Tensor



3D Local Strain State: Strain Tensor

Definition:

$$\varepsilon_{xx} = \lim_{x_0 \to 0} \frac{\Delta x}{x_0}, \quad \varepsilon_{yy} = \lim_{y_0 \to 0} \frac{\Delta y}{y_0}, \quad \varepsilon_{zz} = \lim_{z_0 \to 0} \frac{\Delta z}{z_0}$$

$$\varepsilon_{xy} = \frac{1}{2} \cdot \Delta\gamma, \quad \varepsilon_{xz} = \frac{1}{2} \cdot \Delta\beta, \quad \varepsilon_{yz} = \frac{1}{2} \cdot \Delta\alpha$$

Universal Strain Definition:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i, j = \{x, y, z\}$$

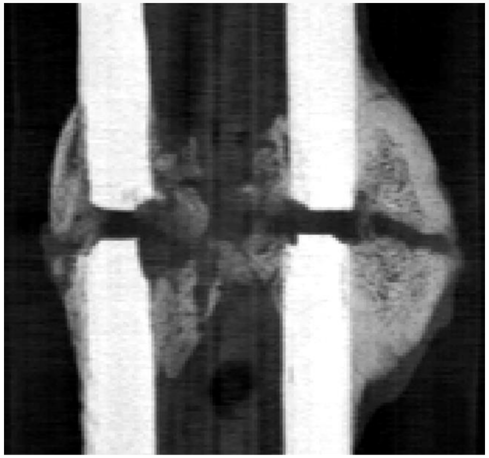
$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

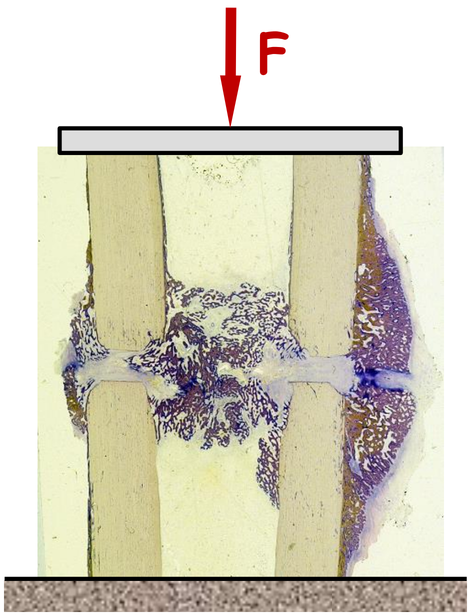
Strain Tensor "

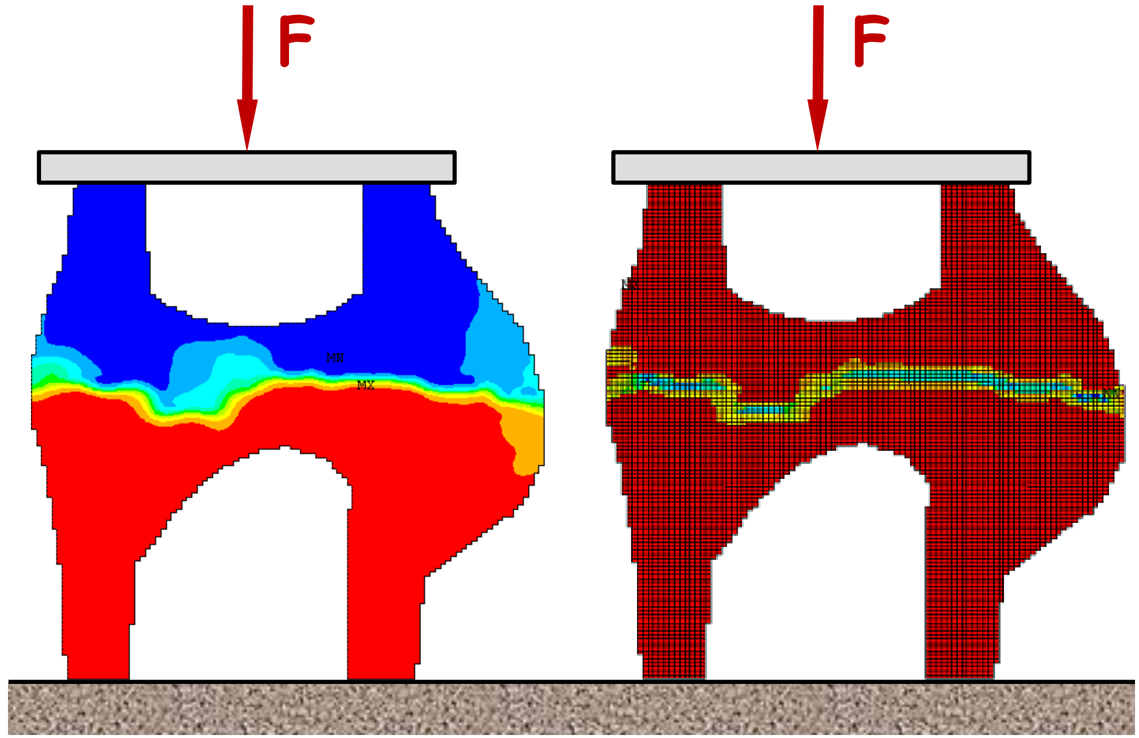
Note to Remember:

Strain is relative change in length (and shape)

Displacement vs. Strain







Displacement u_x

Strain, eps_xx

Anisotropic Properties

Material	E Moduli in MPa	Strength in MPa	Fracture strain in %
Spongy bone			
Vertebra	60 (male) 35 (female)	4,6 (male) 2,7 (female)	6
prox. Femur	240	2.7	2.8
Tibia	450	5...10	2
Bovine	200...2000	10	1,7...3,8
Ovine	400...1500	15	
Cortical bone			
Longitudinal	17000	200	2,5
Transversal	11500	130	