Mechanical Basics of Biomechanics

and

Biomechanical Principles of Bone Structure

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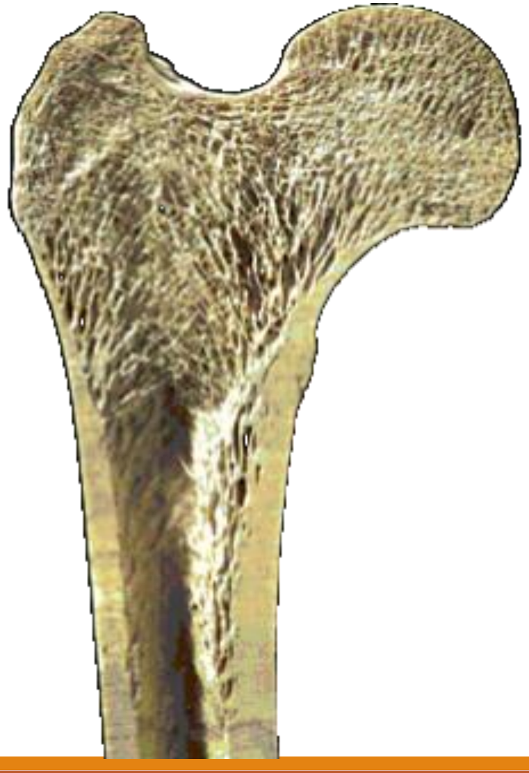
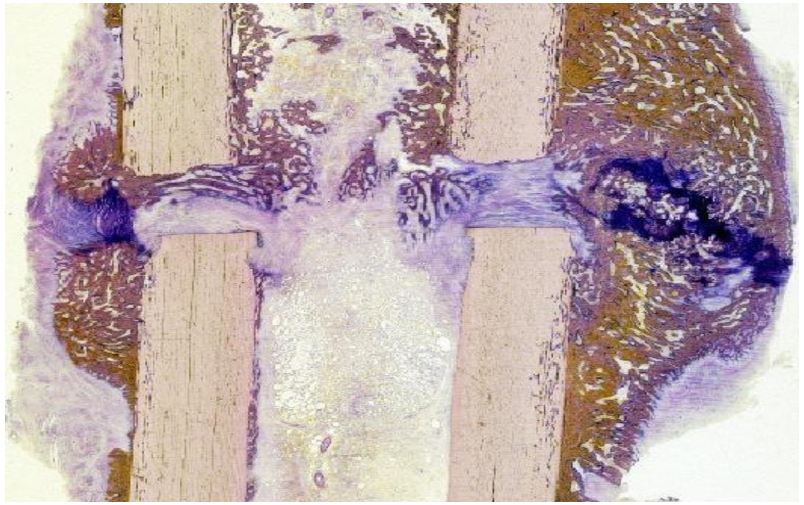
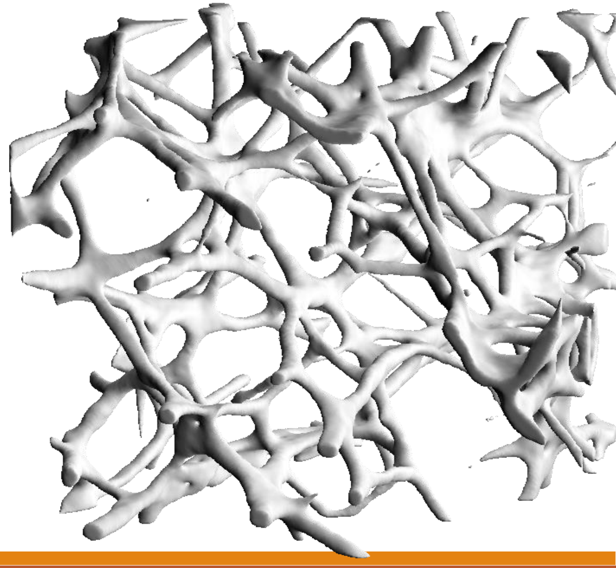
UZWR

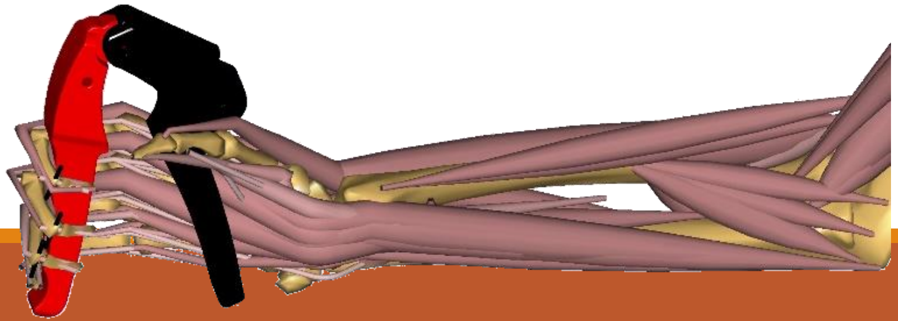






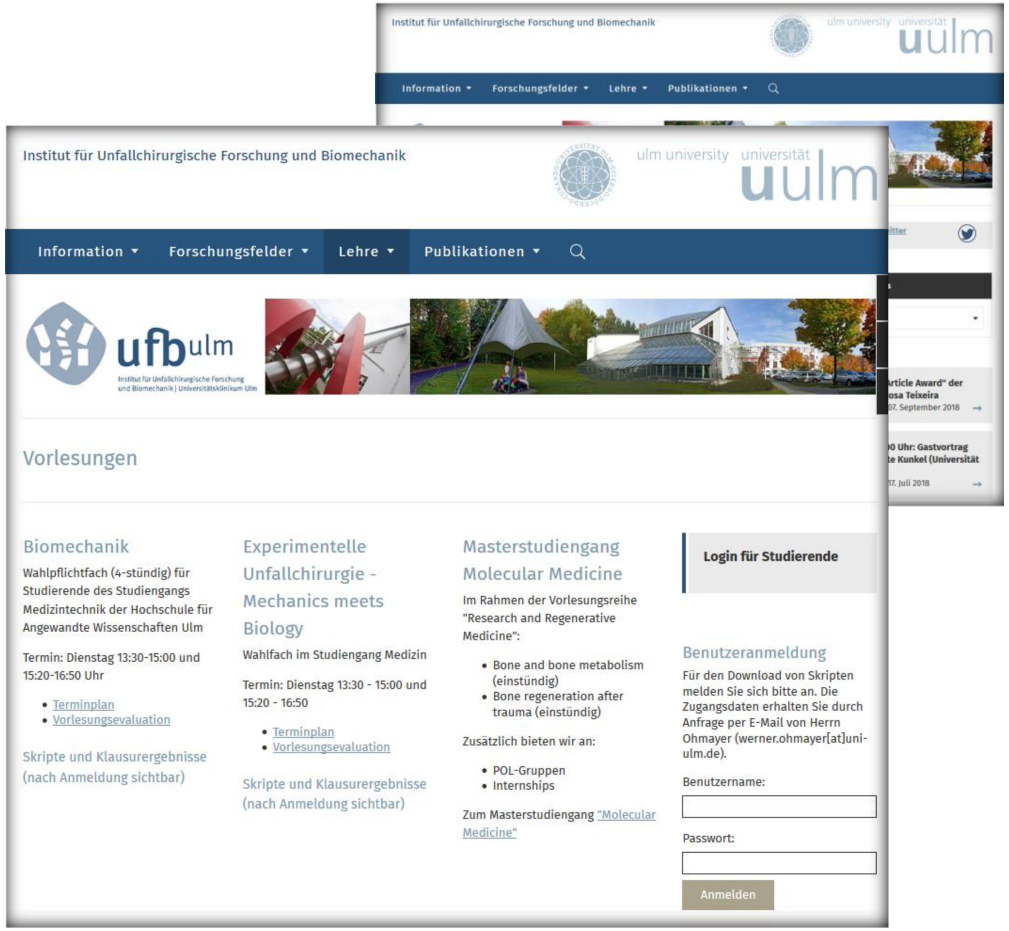
What I do:

- Effects of global force distribution to the fracture healing process
- Local mechanoregulations in metaphyseal
  fractures in healthy, old and osteoporotic humans



www.biomechanics.de

www.uzwr.de



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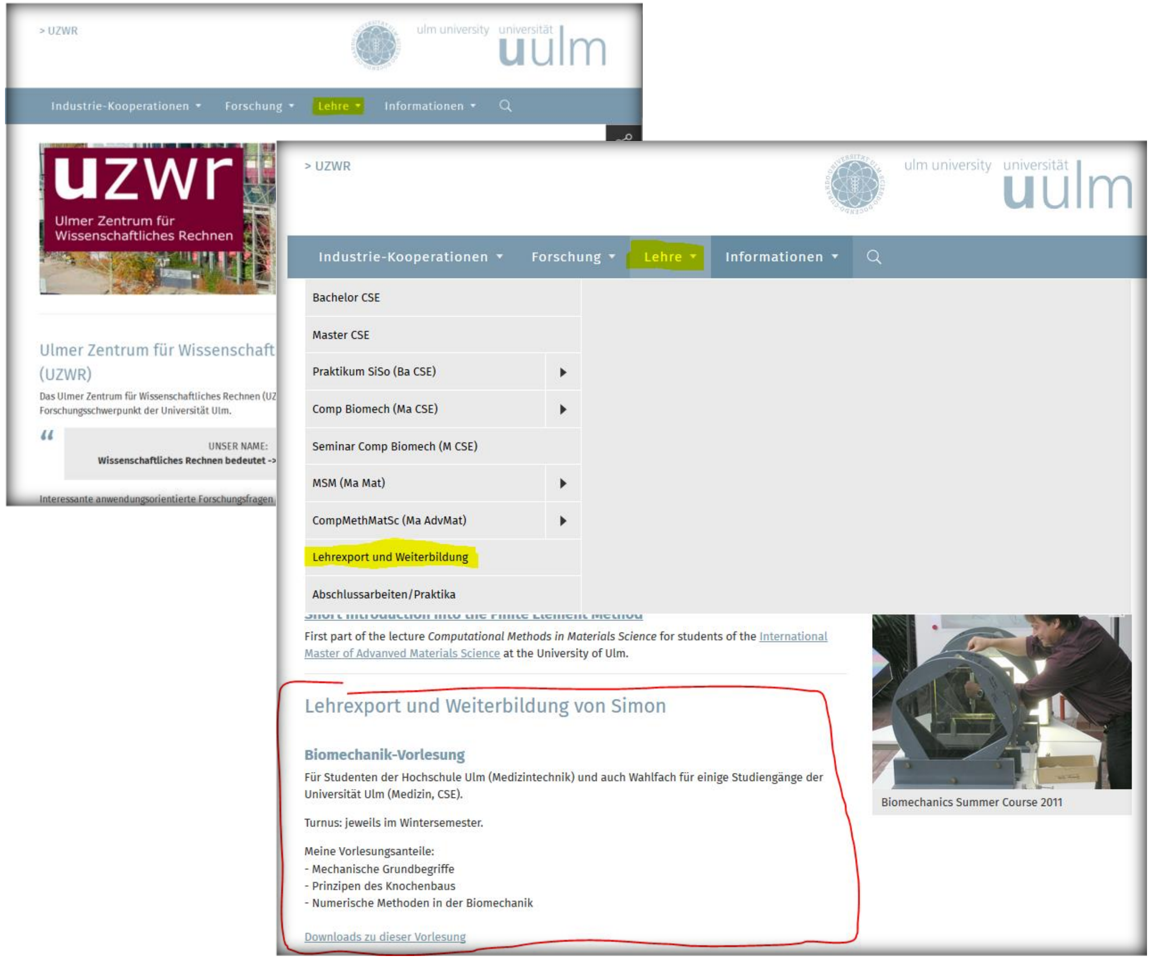
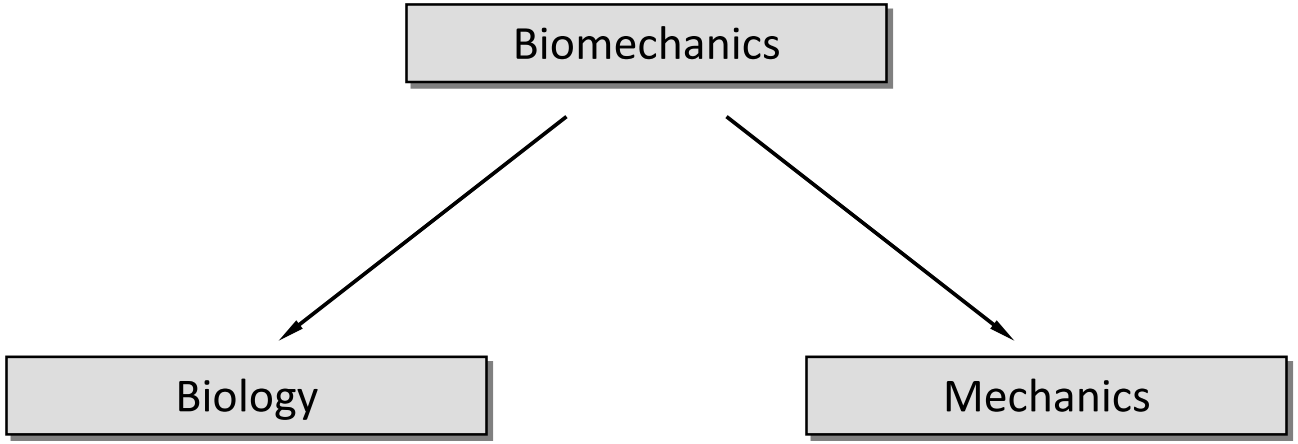


General



The aim of the Lecture:

Brush up mechanical knowledge in an illustrative form.

Outline Lecture 1

1.4. Statics of rigid bodies

1.4.1. the force
1.4.2. The cutting principle (Euler)
1.4.3. Assembling and disassembling of forces
1.4.4. The moment
1.4.5. Moment of a force with respect to a point
1.4.6. Free body diagram
1.4.7. Static equilibrium
1.4.8. Recipe for solving problems of statics
1.4.9. Sample calculation "biceps force"

1.5. Elastostatics / Strength of Materials

1.5.1. The Stress
1.5.2. For example, tension in the muscle
1.5.3. Normal and shear stresses
1.5.4. Strains
1.5.5. Material laws
1.5.6. Simple load cases

1.6. kinematics

1.6.1. Coordinate systems
1.6.2. Translation and rotation
1.6.3. And angle
1.6.4. Speed
1.6.5. Acceleration
1.6.6. Summary

1.7. Kinetics / dynamics

1.7.1. d'Alembert principle
1.7.2. Energy, work, power

Measure, Dimensions, Units

Standard: ISO 31, DIN 1313

Measure = Data · unit

Length $L = 2 \cdot \text{m} = 2\ \text{m}$

{Measure} is the numerical value

[Measure] = Unit

<u>not correct</u>: ~~Length L [m]~~

<u>right</u>: Length L / m

or the length L in m

<u>SI base units (mechanism)</u>:

m (meter), kg (kilogram), s (second)

Unit Systems

base units			derived units					comment
length	*mass*	*time*	*force*	*tension*	*density*	*accel.*	...	
m	**kg**	**sec**					...	SI units
mm		**sec**	**N**				...	Organ Level
							...	tissue-level

[...] means unity of ...

To line 1:

[Force] = [mass] * [length] / [time] ²

[Stress] = [force] / [length] ²

[Density] = [mass] / [length] ³

...

To line 2:

1. Choice: mm
2. Choice: N
3. Choice: sec

[Mass] =?

[Force] = [mass] * [length] / [time] ²

[Mass] = [force] * [time] ² / [length]

= (Kg * m / sec²) * sec² / mm

= 1000 kg

= t

The Force

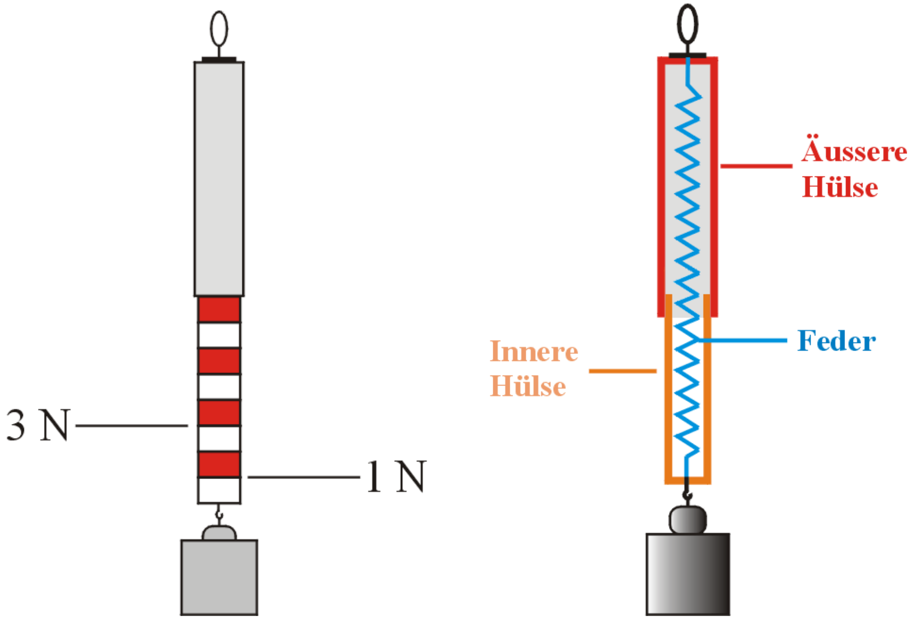
- The force-term is well known from everyday life: driving force, muscleforce, laborforce, ...
- but actually axiomatic, without strict definition
- „Force" is an invention, not a discovery
- Forces can not be measured directly.

Newton's axiom:

Force = Mass · Acceleration or $F = m \cdot a$

Remember to:

The force is the cause of an acceleration (change in velocity) or deformation (strain) of a body.

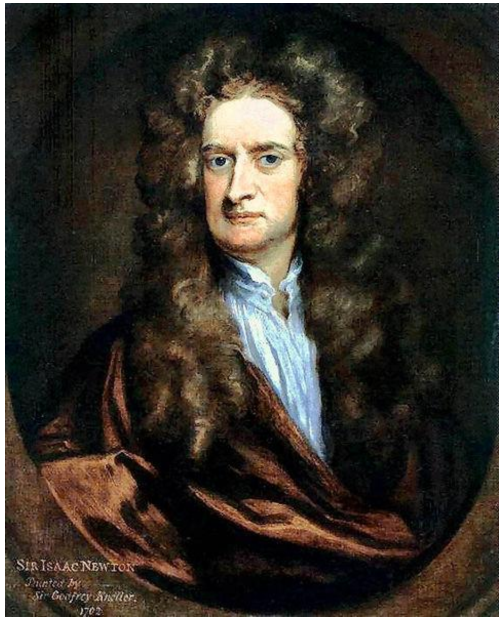


The Unit of Force

Newton

N = kg·m/ s^2

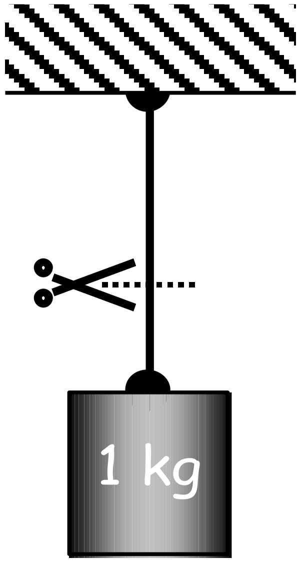
$$F_G = m \cdot g$$
$$= 0{,}102\,\text{kg} \cdot 9{,}81\,\text{N/kg}$$
$$= 1\,\text{N}$$

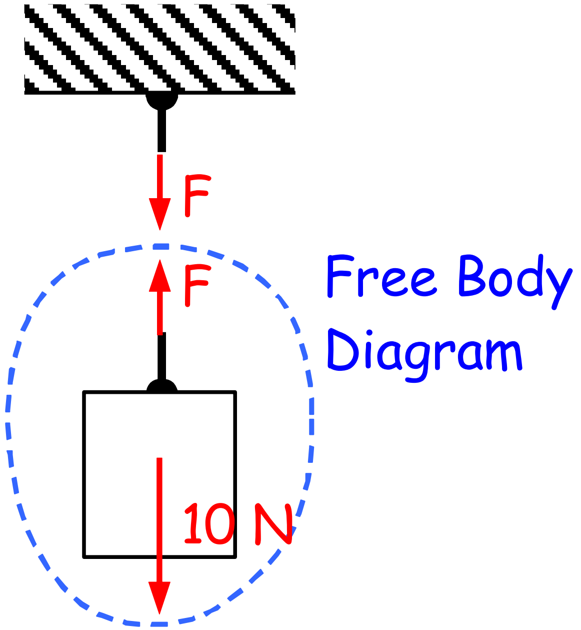


Remember to:

Weight of a chocolate bar ≈ 1 Newton

Cutting principle (Euler) and free-body diagram





Remember to:

<u>First</u> to cut <u>then</u> Enter forces and moments.

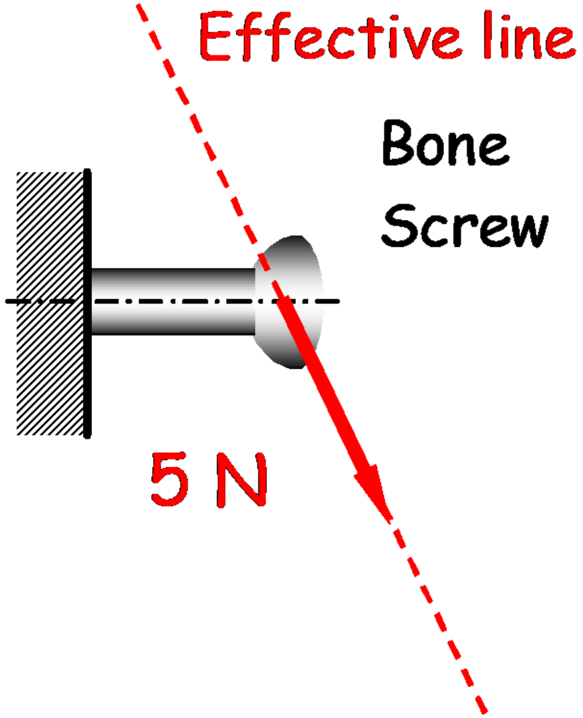
Free body diagram = completely cutted subsystem

Presenting Forces

... with arrows

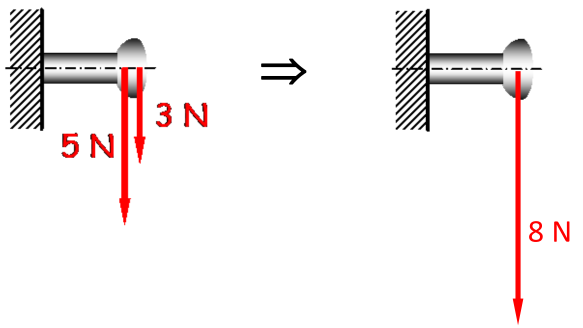
forces are *vectorial* sizes

- **amount**
- **direction**
- **sense of direction**

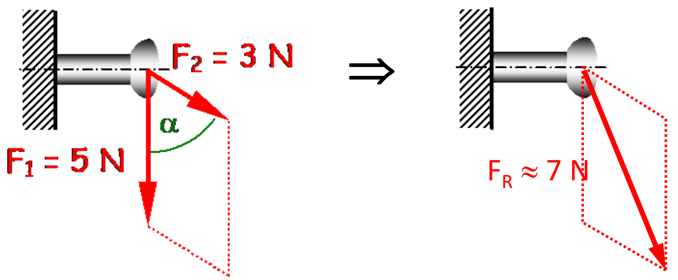


Assembling and disassembling of forces

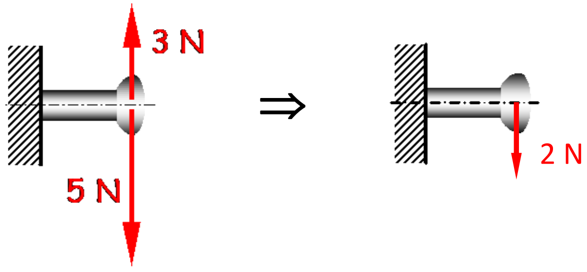
Addition of the amounts



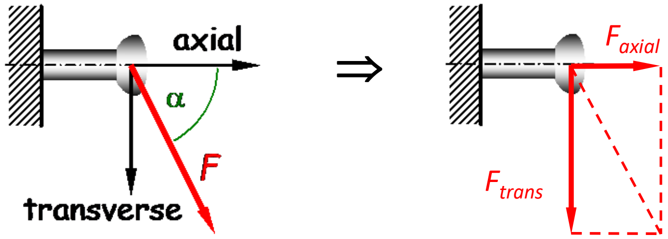
Vector addition



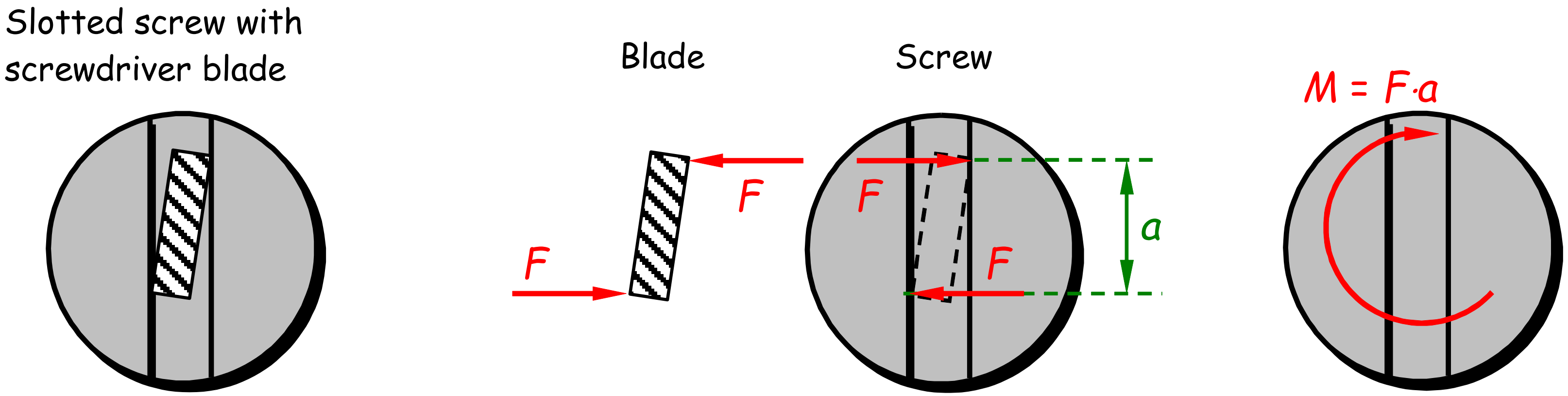
Subtracting the amounts



Breaking it down into components



The Moment



Couple of Forces (F. a)

Moment M

Remember to:

A moment is the cause of a rotational acceleration (change in velocity) or a (rotary) deformation (torsion, bending) of a body.

To think:

Moment is „Roational Force"

The Moment

Unit of the moment

Newton-meters

$N{\cdot}m = kg{\cdot}m^2/ s^2$

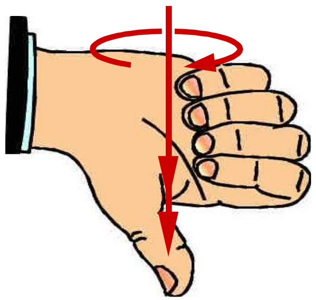
Representation of moments

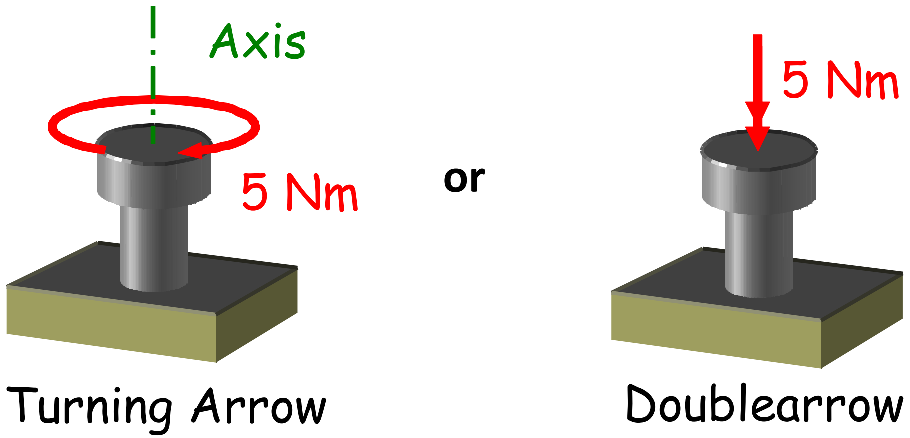
... with turning arrows or double arrows

moments *vectorial* sizes

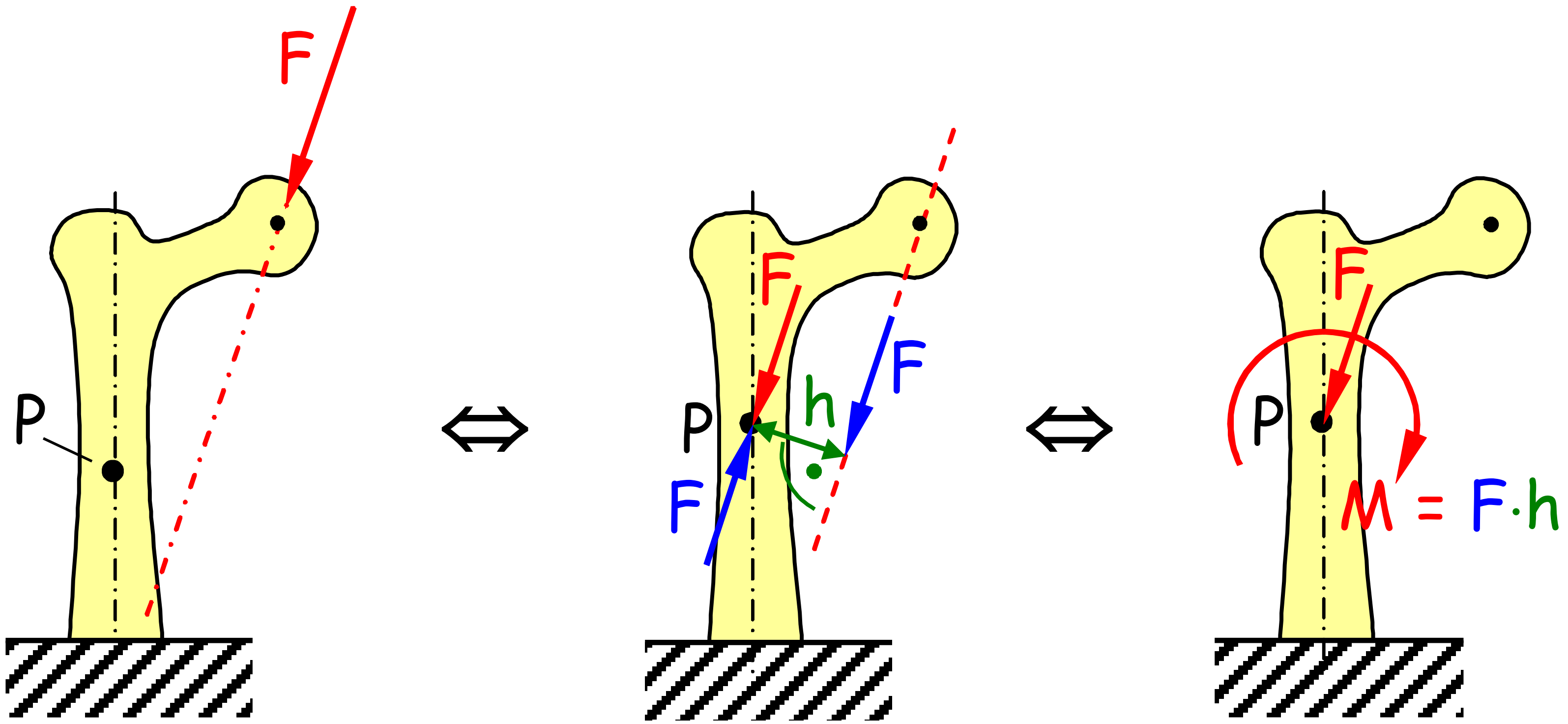
- **amount**
- **direction**
- **sense of direction**

Right-hand rule:





Moment of a Force with respect to a Point P



Remember to:

Moment = force times lever arm

(Lever arm perpendicular to the line of action)

Static Equilibrium

Important:

Balance only to "free body diagrams"

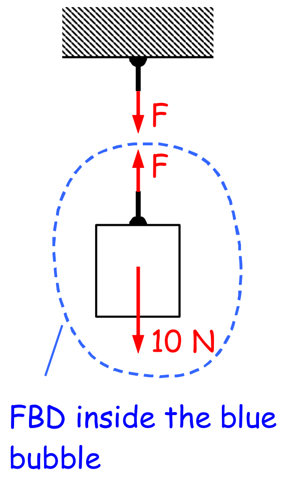
For a flat (2D) apply problem three equations:

$$\text{Summe aller Kräfte in x-Richtung:} \quad F_{1,x} + F_{2,x} + \ldots \overset{!}{=} 0,$$

$$\text{Summe aller Kräfte in y-Richtung:} \quad F_{1,y} + F_{2,y} + \ldots \overset{!}{=} 0,$$

$$\text{Summe aller Momente bezüglich P:} \quad M_{1,z}^{P} + M_{2,z}^{P} + \ldots \overset{!}{=} 0.$$

(For a spatial (3D) apply problem six equations)



Solution Rsecipe

Step 1: Modeling, Generate a replacement model (sketch geometry, loads, restraints). Omitting unimportant things. The "real system" must be abstracted.

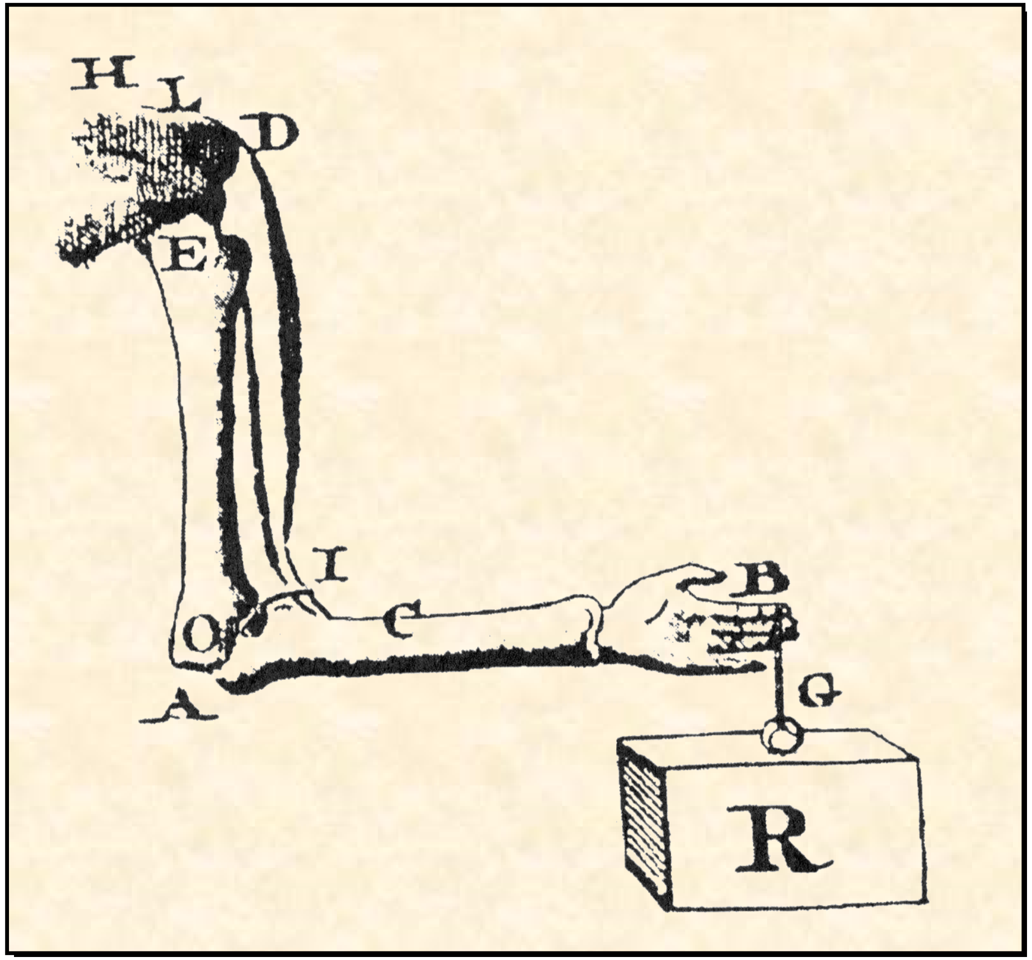
Step 2: To cut, Free-body diagrams. System slice, add internal forces and internal moments,

Step 3: Balance, write force and moment equilibria for free body.

Step 4: Equations to solve,

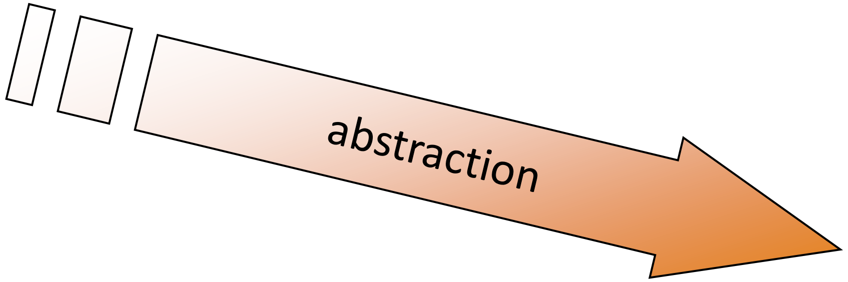
Step 5: Result interpret, Verify, compare with experiment; Check validity.

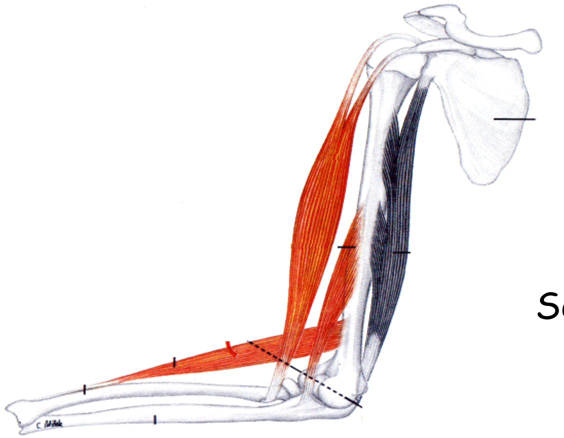
Classical Calculation: „Biceps Force"

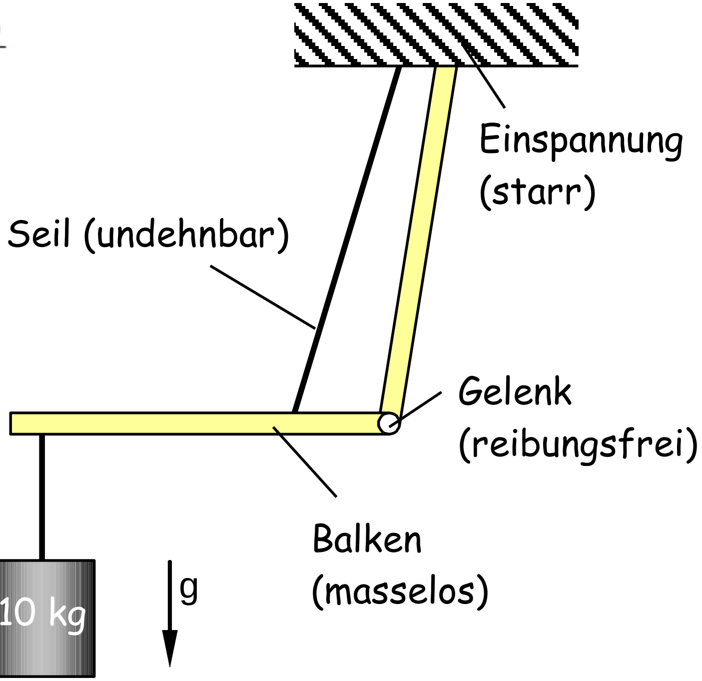


Out:
"De Motu Animalium"
from
GA Borelli
(1608-1679)

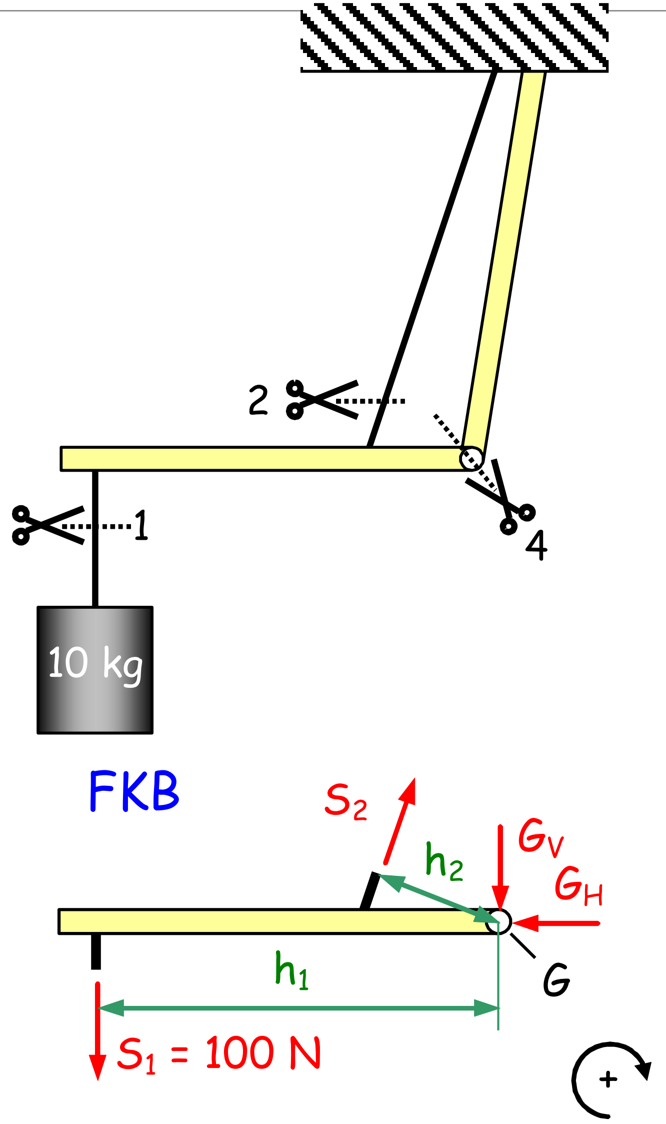
Step 1: Modeling







Step 2: Cutting and Free-Body Diagrams



step 3: balance

step 4: Solve equations

Sum of all moments with respect to Point G = 0

$$-S_1 \cdot h_1 + S_2 \cdot h_2 \overset{!}{=} 0$$

$$-100\,\text{N} \cdot 35\,\text{cm} + S_2 \cdot 5\,\text{cm} \overset{!}{=} 0$$

$$\Rightarrow S_2 = 100\,\text{N} \cdot \frac{35\,\text{cm}}{5\,\text{cm}} = \underline{\underline{700\,\text{N}}}$$

This is seven times the load!

Elastostatics

STRENGTH OF MATERIALS

Stress

Photos: Lutz Dürselen

to Remember:

Stress = "smeared" cutting force,

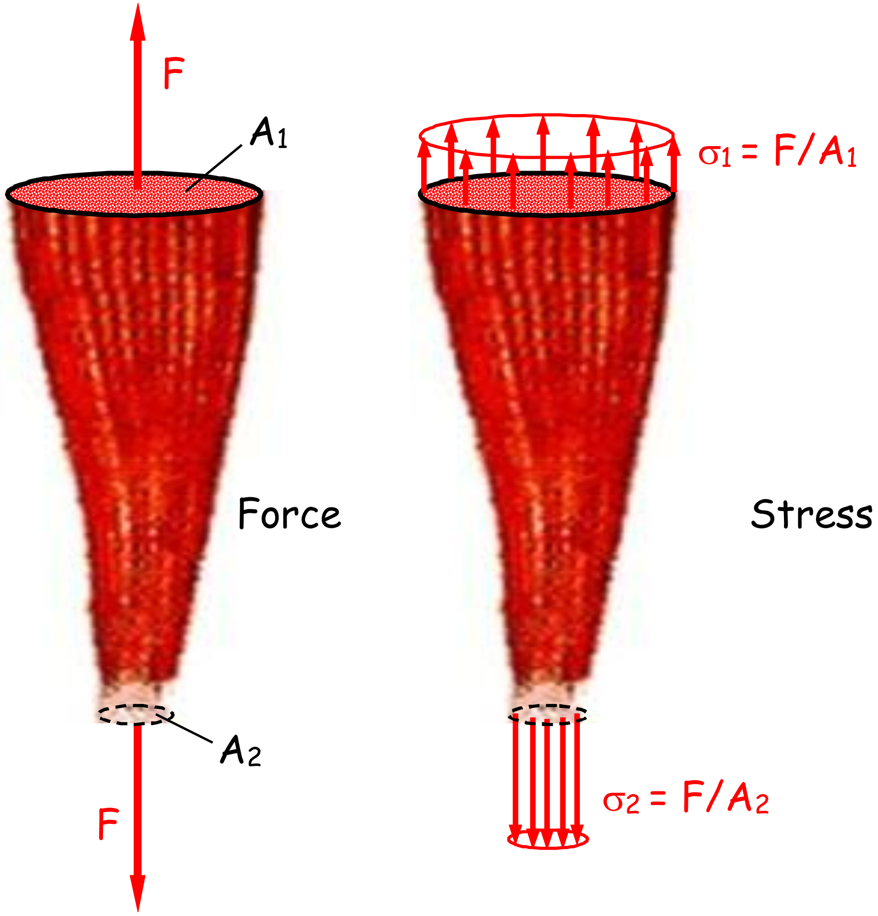
Stress = force per area or $\sigma = F / A$

Unit of the Stress

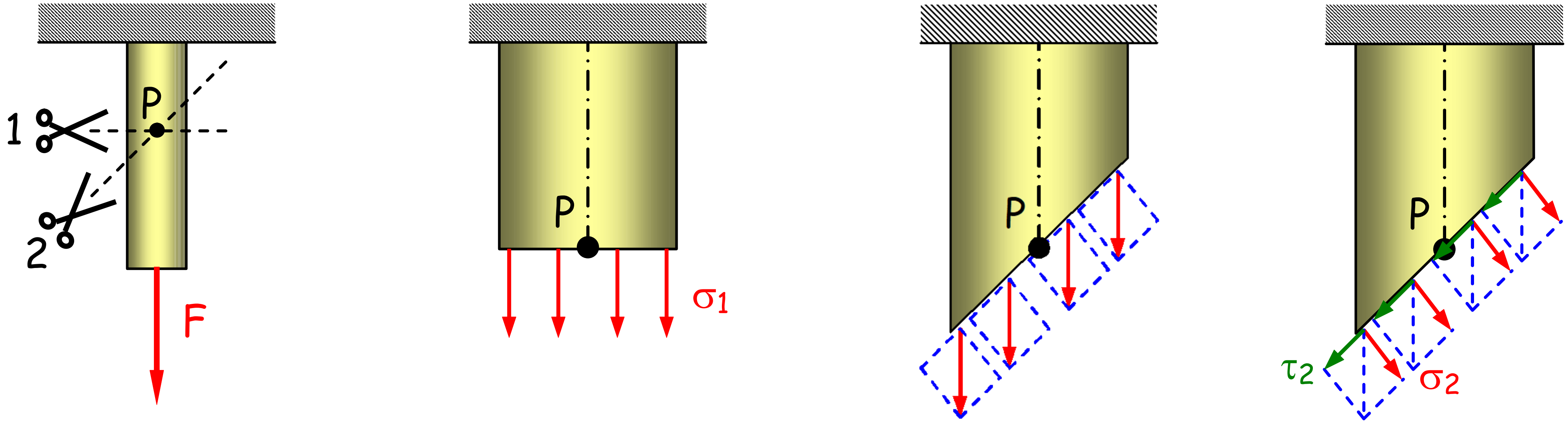
Mega-Pascal 1 Mpa = 1 N / mm^2

Pascal: 1 Pa = 1 N / m^2

Example: Tension in the muscle



Normal and Shear Stresses



Tensile bar

Cut 1:
→ Normal stresses σ_1

Cut 2:
→ Normal σ_2 and
shear stresses τ_2

Normal and Shear Stresses

Remember to:

First cut, then the type and magnitude of the stress.

normal stresses (Tensile and Compression) Perpendicular to the sectional area

shear stresses Parallel to the cut surface.

General (3D) Stress State ... !

... at one point of the body:

- **<u>Three</u> Stress components in a section (Normal., 2x Shear.)**

times

- **<u>Three</u> Sections (eg frontal, sagittal, transverse)**

equal

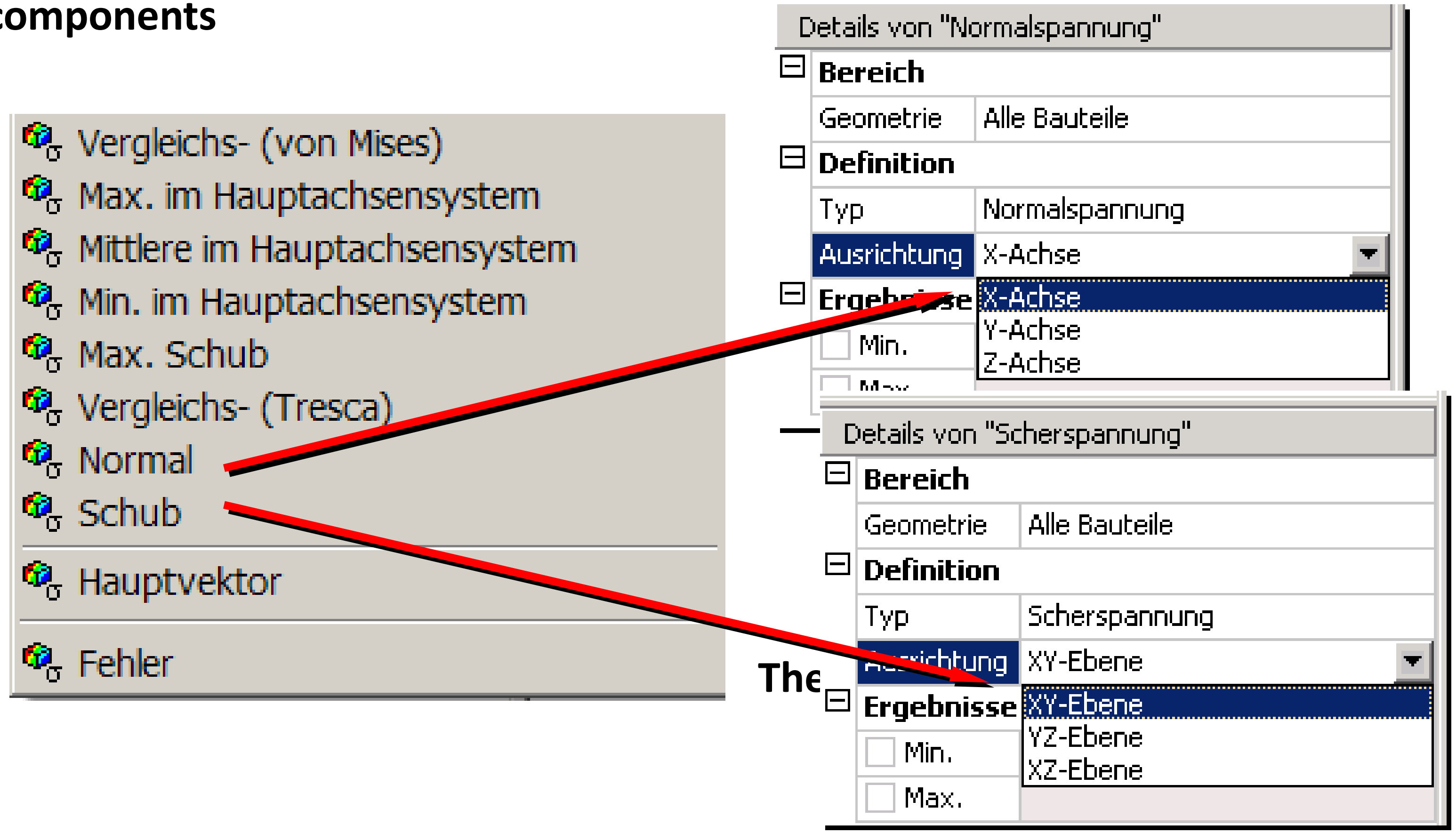
- **<u>nine</u> Stress components that characterize the full 3D stress state at one point in the body.**
- **<u>six</u> Components thereof are independent („Equality of Shearstresses")**

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$

The "stress tensor"

General State of Stress

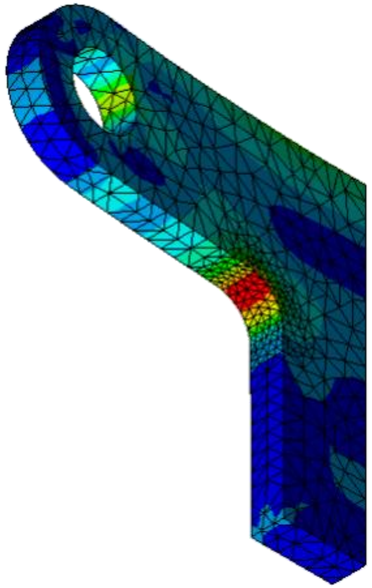
<u>six</u> components



The

Representation of the Stress State

- **Problem: If you want to make colorful pictures, you have to decide for a component.**
- **But which should we take?**
- **Can be used instead of a single well "mixtures" of the component.**
- **So-called „Invariants" are nothing more where the same comes out regardless of the orientation of the coordinate system as particularly "smart" mixtures „Principal Stresses," „Von-Mises-Stress "," Hydrostatic Stress Component "," Octahedral Shear Stress ", ...**



- Vergleichs- (von Mises)
- Max. im Hauptachsensystem
- Mittlere im Hauptachsensystem
- Min. im Hauptachsensystem
- Max. Schub
- Vergleichs- (Tresca)
- Normal
- Schub

$$\sigma_{Mises} = \sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\,\sigma_{yy} - \sigma_{xx}\,\sigma_{zz} - \sigma_{yy}\,\sigma_{zz} + 3\,\tau_{xy}^2 + 3\,\tau_{xz}^2 + 3\,\tau_{yz}^2}$$

Strains

DETAILS

- **Rope type:** Single rope
- **Diameter:** 10.5 mm
- **Impregnation:** without
- **Weight:** 72 g per meter
- **Impact force:** 9.6 kN
- **Num. falls:** 10
- **Sheath slippage:** 0 mm
- **Strain static:** 7.07%
- **Strain dynamic:** 32%
- **knotability:** 0.7
- **Colour:** mix

Remember to:

Strain = relative length change (angle change)

Strains

Definition of strain

$$\text{Strain} \coloneqq \frac{\text{Elongation}}{\text{Original Length}}$$

$$\varepsilon \coloneqq \frac{\Delta \mathrm{L}}{\mathrm{L}_0}$$

Unit of strain

Without unit. so eg:

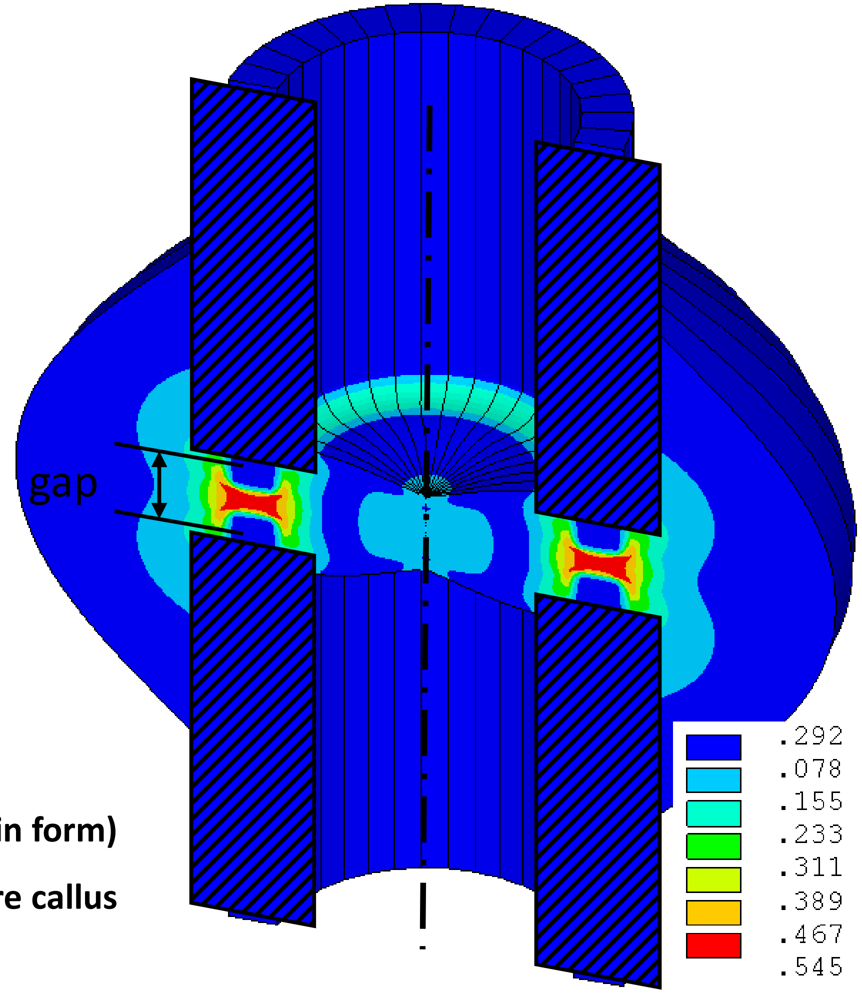
1

1/100 =%

1 / 1,000,000 = με (micro strain) = 0.1%

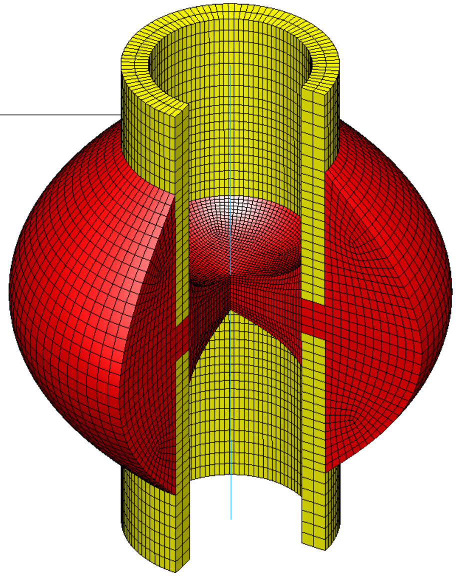
Strains

- Global outer elongation
- Local internal expansions



Elongation (change in form)

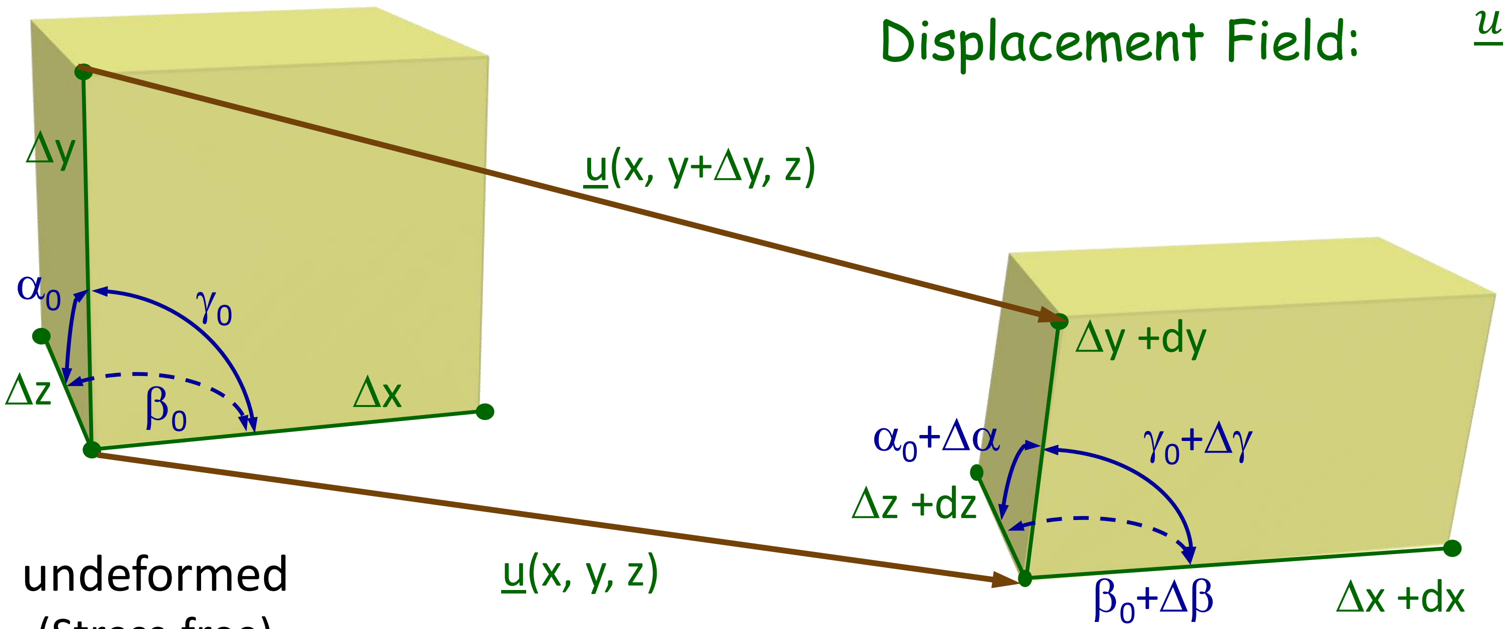
the fracture callus



Defining the local strain state

Infinitesimal test volume ΔV = Δx • Δy • Δz

Displacement Field: $\underline{u} = \begin{pmatrix} u_x(x,y,z) \\ u_y(x,y,z) \\ u_z(x,y,z) \end{pmatrix}$



undeformed
(Stress-free)

deformed
(Stress on all surfaces)

Defining the local strain state

!

$$\varepsilon_{xx} = \lim_{\Delta x \to 0} \frac{dx}{\Delta x}, \quad \varepsilon_{yy} = \lim_{\Delta y \to 0} \frac{dy}{\Delta y}, \quad \varepsilon_{zz} = \lim_{\Delta z \to 0} \frac{dz}{\Delta z}$$

$$\varepsilon_{xy} = \frac{1}{2} \cdot \Delta\gamma, \quad \varepsilon_{xz} = \frac{1}{2} \cdot \Delta\beta, \quad \varepsilon_{yz} = \frac{1}{2} \cdot \Delta\alpha$$

Normal strain based on displacement field u

$$\varepsilon_{xx} = \lim_{\Delta x \to 0} \frac{dx}{\Delta x} = \lim_{\Delta x \to 0} \frac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x} = u_{x,x}$$

Universal definition of normal AND shear strain components

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i, j = \{x, y, z\}$$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix}$$

The "strain tensor"

Material Laws

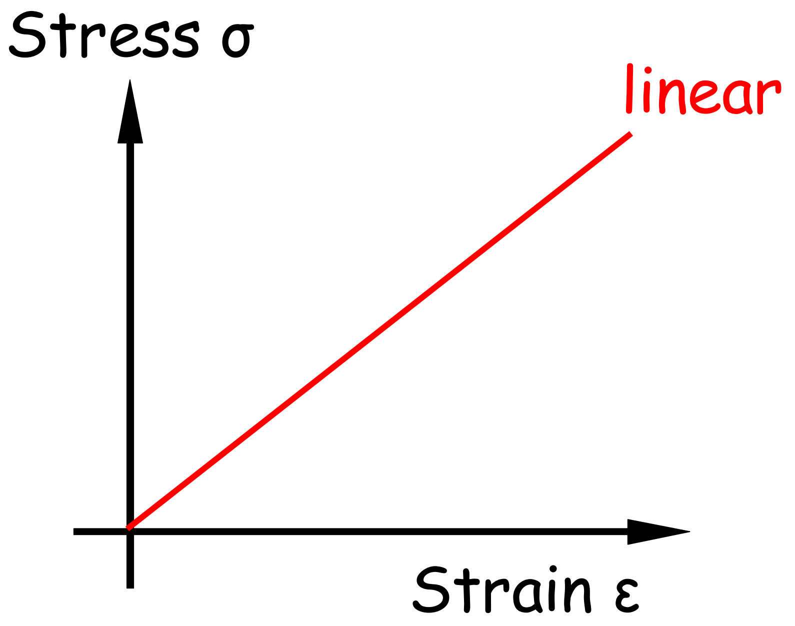
... linking stresses and strains together

Linear Materials Law:

$$\sigma = E \cdot \varepsilon$$

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{E}}}} \cdot \underline{\underline{\varepsilon}}$$

$$\sigma_{ij} = E_{ijkl} \cdot \varepsilon_{kl}$$



- Fully occupied tensor 4th stage for three dimensions 81 parameters (9x9)
- Equality of associated shear stresses
 (Boltzmann continua) and shear strains 36 parameters (6x6)

!

$$\underline{\sigma} = \underline{\underline{E}} \cdot \underline{\varepsilon}$$

- Maxwell's reciprocity theorem (Betty‘s theorem) 21 parameter
- Orthotrop (trabecular bone) 9 parameter
- Transverse Isotropic (cortical bone) 5 parameter
- isotropic 2 parameter

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix} = \frac{E}{(1+v)\cdot(1-2v)} \cdot \begin{bmatrix} (1-v) & v & v & 0 & 0 & 0 \\ & (1-v) & v & 0 & 0 & 0 \\ & & (1-v) & 0 & 0 & 0 \\ & & & \frac{(1-2v)}{2} & 0 & 0 \\ & & & & \frac{(1-2v)}{2} & 0 \\ \text{sym} & & & & & \frac{(1-2v)}{2} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}$$

E - elasticity modulus [Young's modulus]

ν - Poisson's ratio (0 ... 0.3 ... 0.5)

to Remember:

A linear-elastic, isotropic material behavior is <u>two</u> Material parameters in:

eg: E and ν

A general anisotropic material law 21 has material parameters.

<u>Two from:</u>

E **- modulus of elasticity, Modulus of elasticity [Young's modulus]**

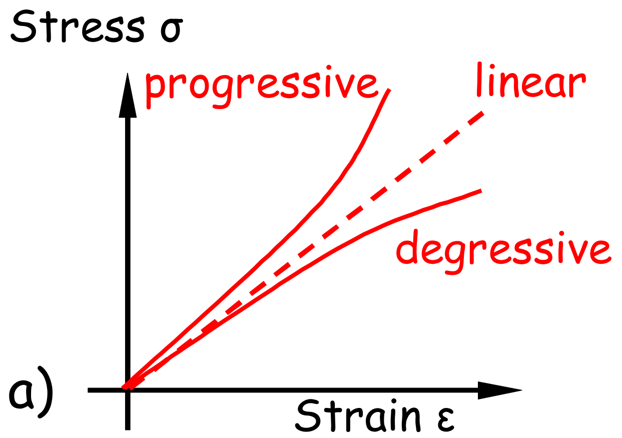
ν - Poisson's ratio [Poisson's ratio] (0 ... <u>0.3</u> ... 0.5)

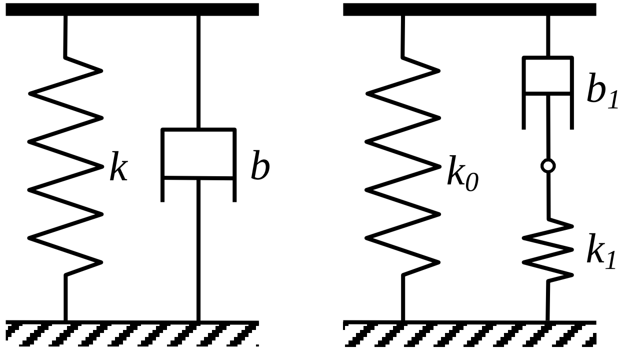
G - shear modulus [Shear modulus]

K - bulk modulus [Bulk modulus]

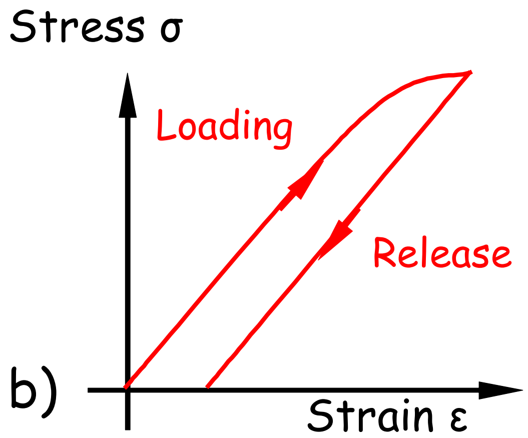
μ, λ - Lame constants [Lame Constant]

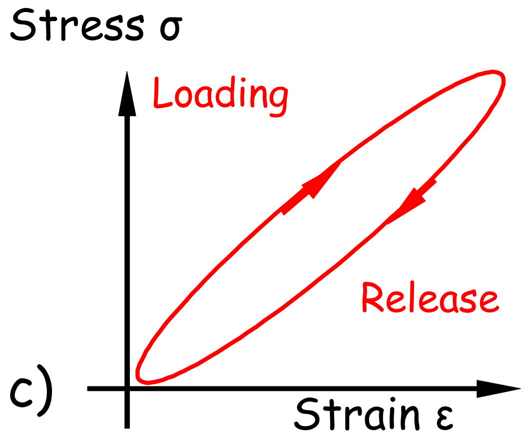
Complex Material Laws:



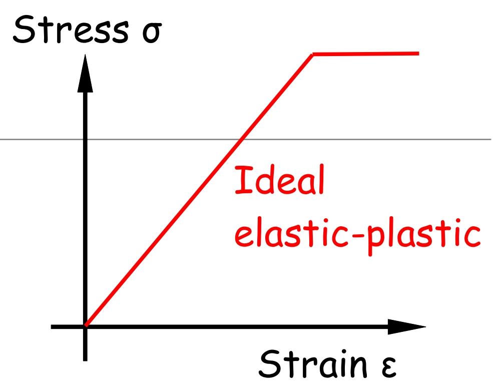


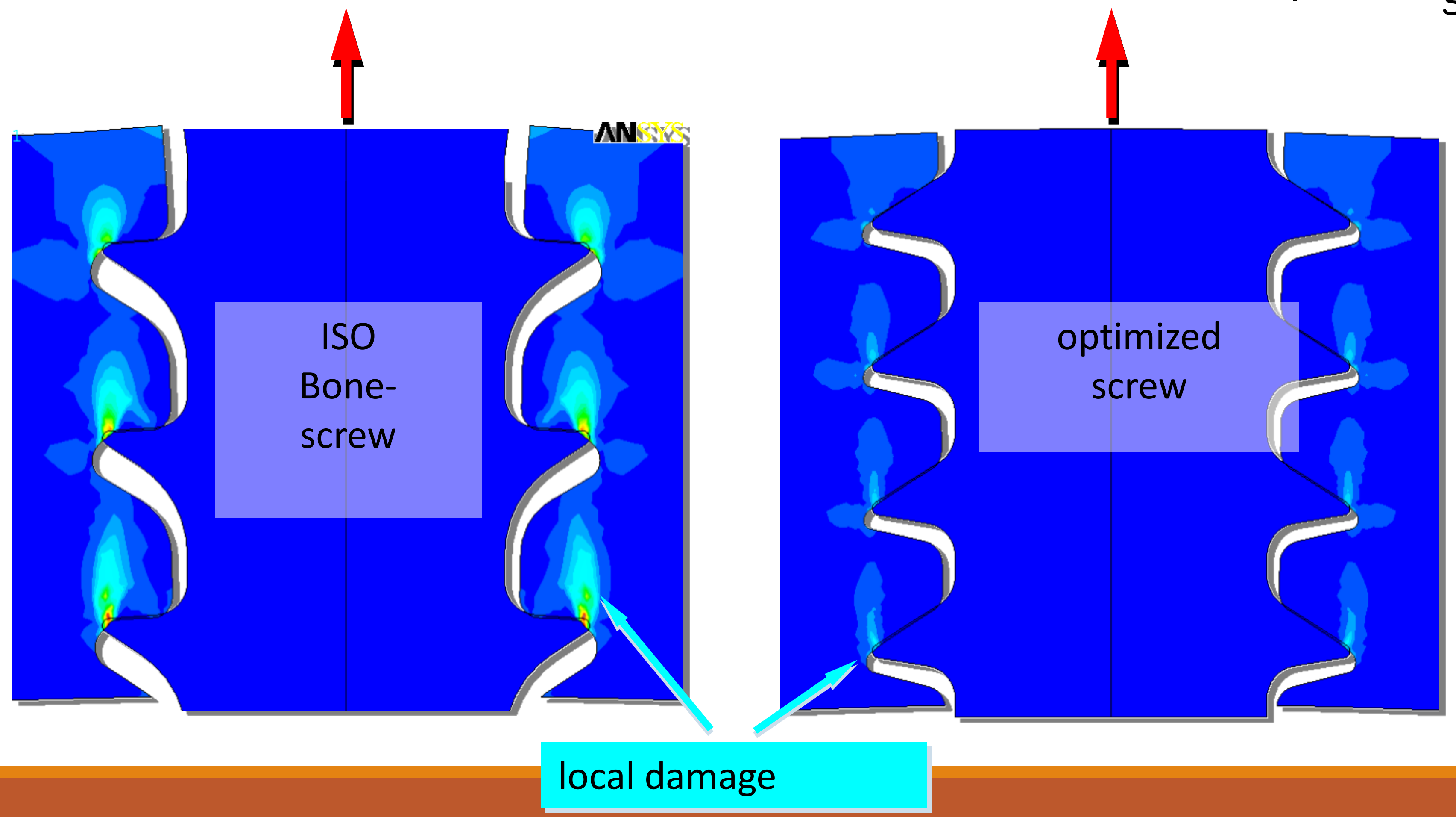
- **Nonlinearly**
- **Non-elastic**
- **anisotropic**
- **Viscoelastic, type: internal damping**
- **Viscoelastic, type: memory effect**



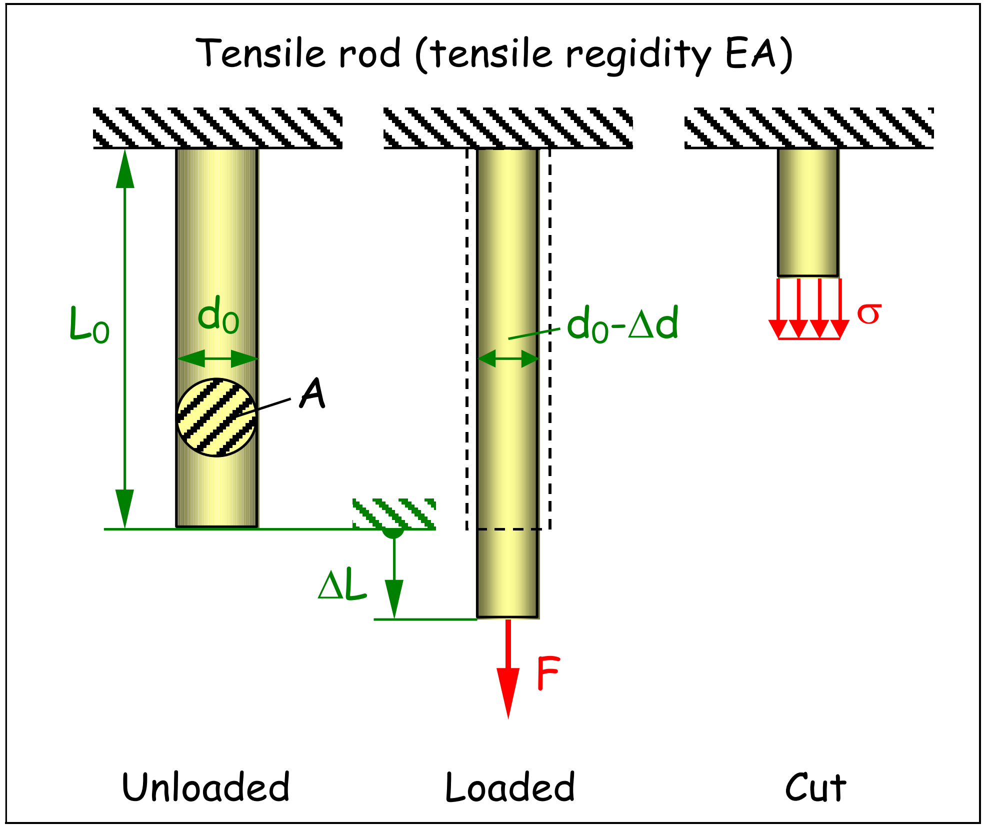


Plastic Strains





Simple load cases: 1. Tension and Compresion



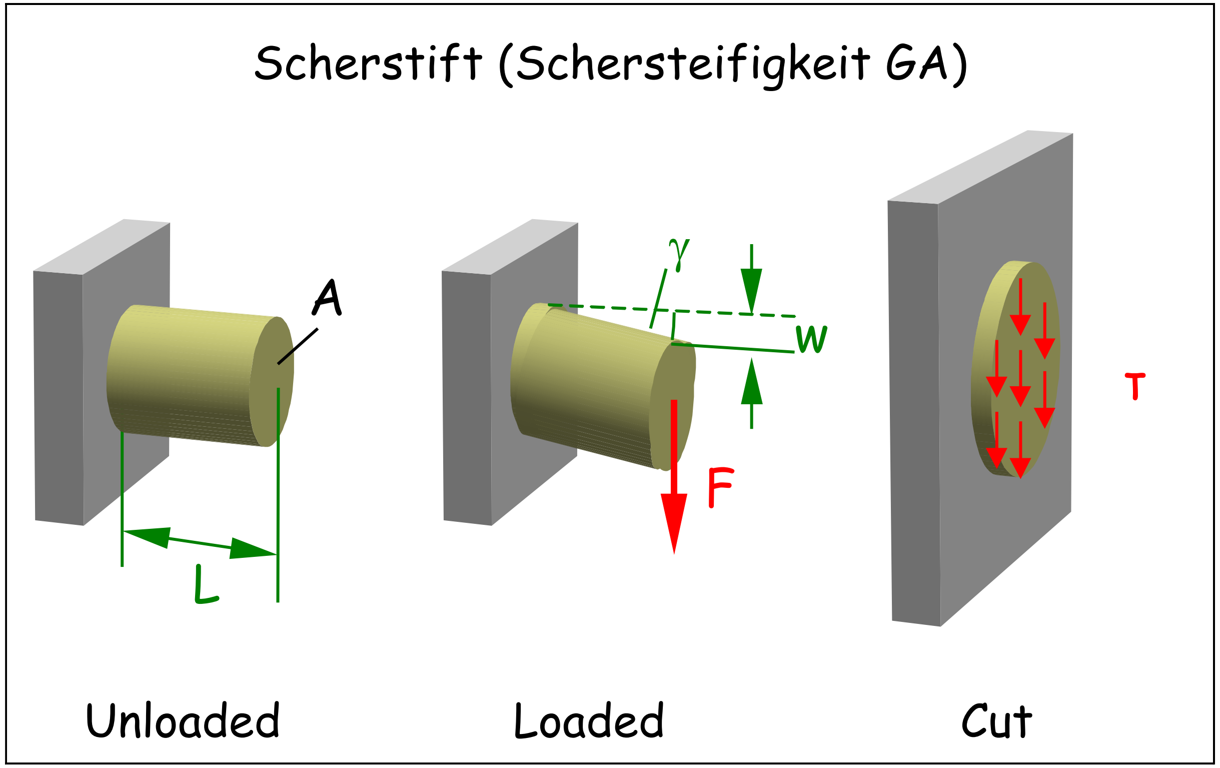
Global behavior, stiffness

$$F = \frac{EA}{L_0}\Delta L, \quad k = \frac{EA}{L_0}$$

Stresses in transverse cut

$$\sigma = \frac{F}{A}$$

2. Shear



Global behavior, stiffness

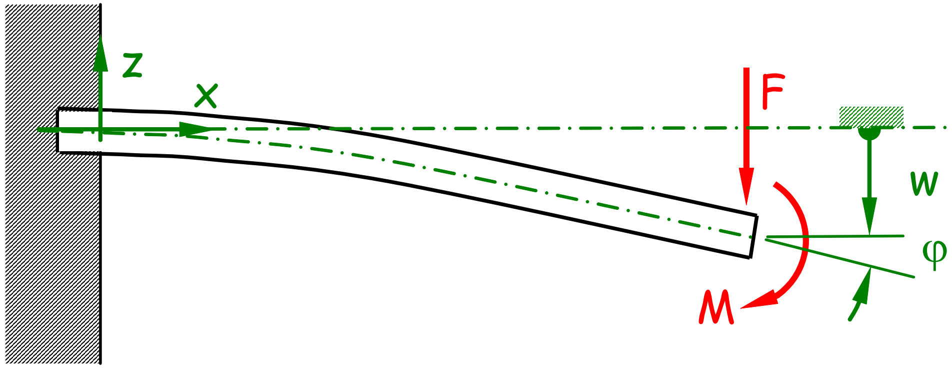
$$F = \frac{GA}{L} w, \quad k = \frac{GA}{L}$$

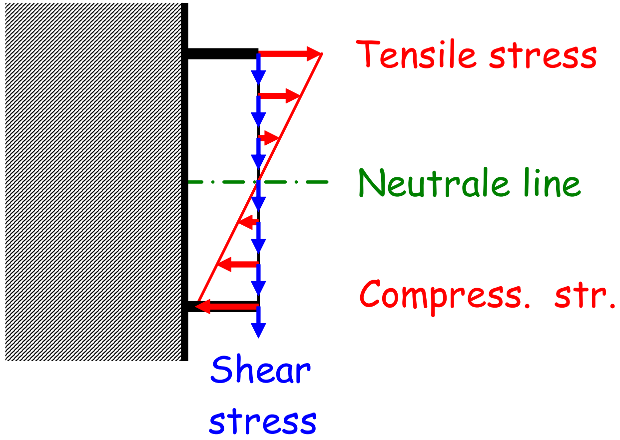
Stresses in transverse cut

$$\tau = \frac{F}{A}$$

3. Bending (cantilever)

Cantilever (bending stiffness EI_a, Length L) section





Global behavior, compliance

$$w = \frac{L^3}{3EI_a} F + \frac{L^2}{2EI_a} M,$$

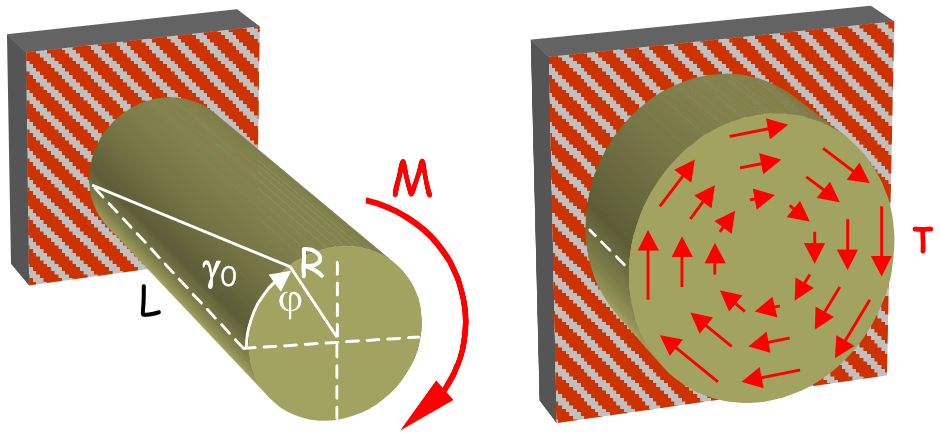
$$\varphi = \frac{L^2}{2EI_a} F + \frac{L}{EI_a} M.$$

Local stress in transverse cut:
(normal stress)

$$\sigma_{xx}(x,z) = \frac{M + F(x-l)}{I_a} z$$

4. Torsion

Torsional rod (torsional regidity GI_T)



Length L, radius r

Cut

Global behavior, stiffness

$$M = \frac{GI_T}{L}\varphi, \quad c = \frac{GI_T}{L}$$

Stresses in transverse cut

$$\tau = \frac{M}{I_T}\rho$$

$$\rho = \text{Distance from Center}$$

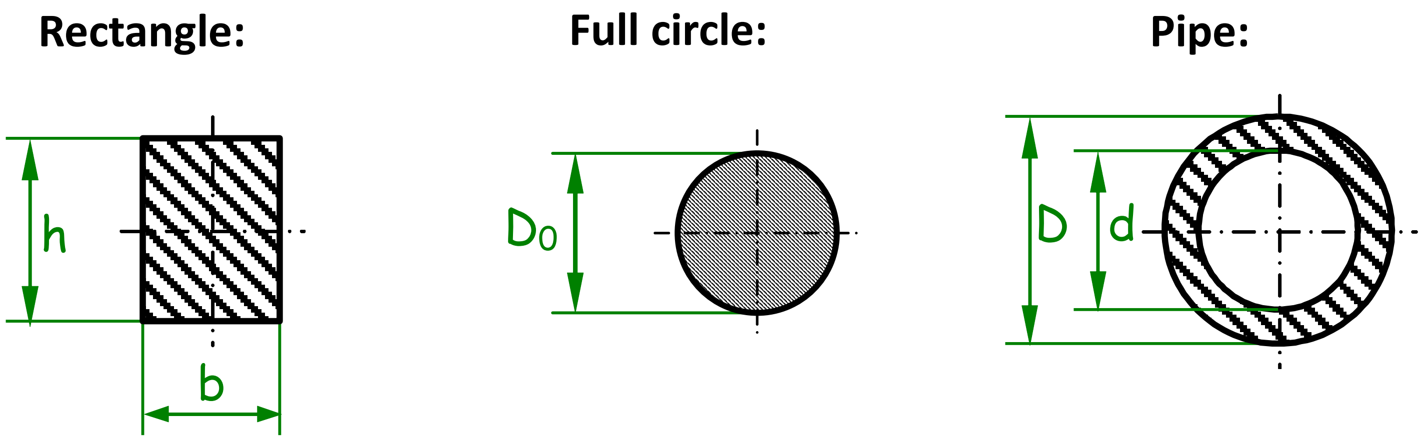
to Remember:

The bones has a favorable (material saving) shape in torsional and bending stresses.

2nd Moment of Area
(Formerly "Moment of inertia") [Second Moment of Area]

$$I_{yy} := \int_A r_z^2 dA$$

!



Axial Second moment of area (bending)

$$I_a = \frac{b \cdot h^3}{12} \qquad I_a = \frac{\pi}{64} D_0^4 \qquad I_a = \frac{\pi}{64}(D^4 - d^4)$$

Polar moment of area of the second degree (torsion)

$$I_T = I_P = \frac{\pi}{32} D_0^4 \qquad I_T = I_p = \frac{\pi}{32}(D^4 - d^4)$$

Kinematics and Dynamics

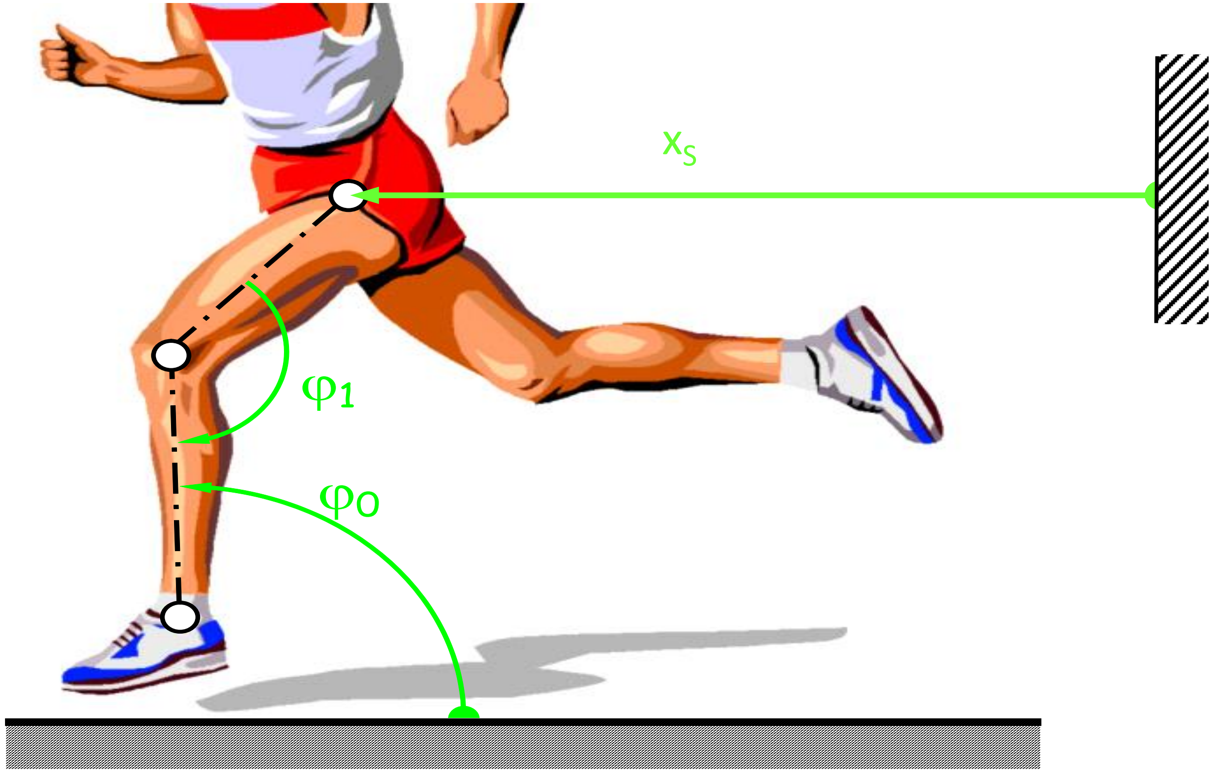
Kinematics

- Describes and analyzes movements without considering forces.
- For rigid bodies a final amount of coordinates satisfy a description.
- Coordinates describe the position of the body at any time.
- In biomechanics: *Gait analysis, joint kinematics*,

to Remember:

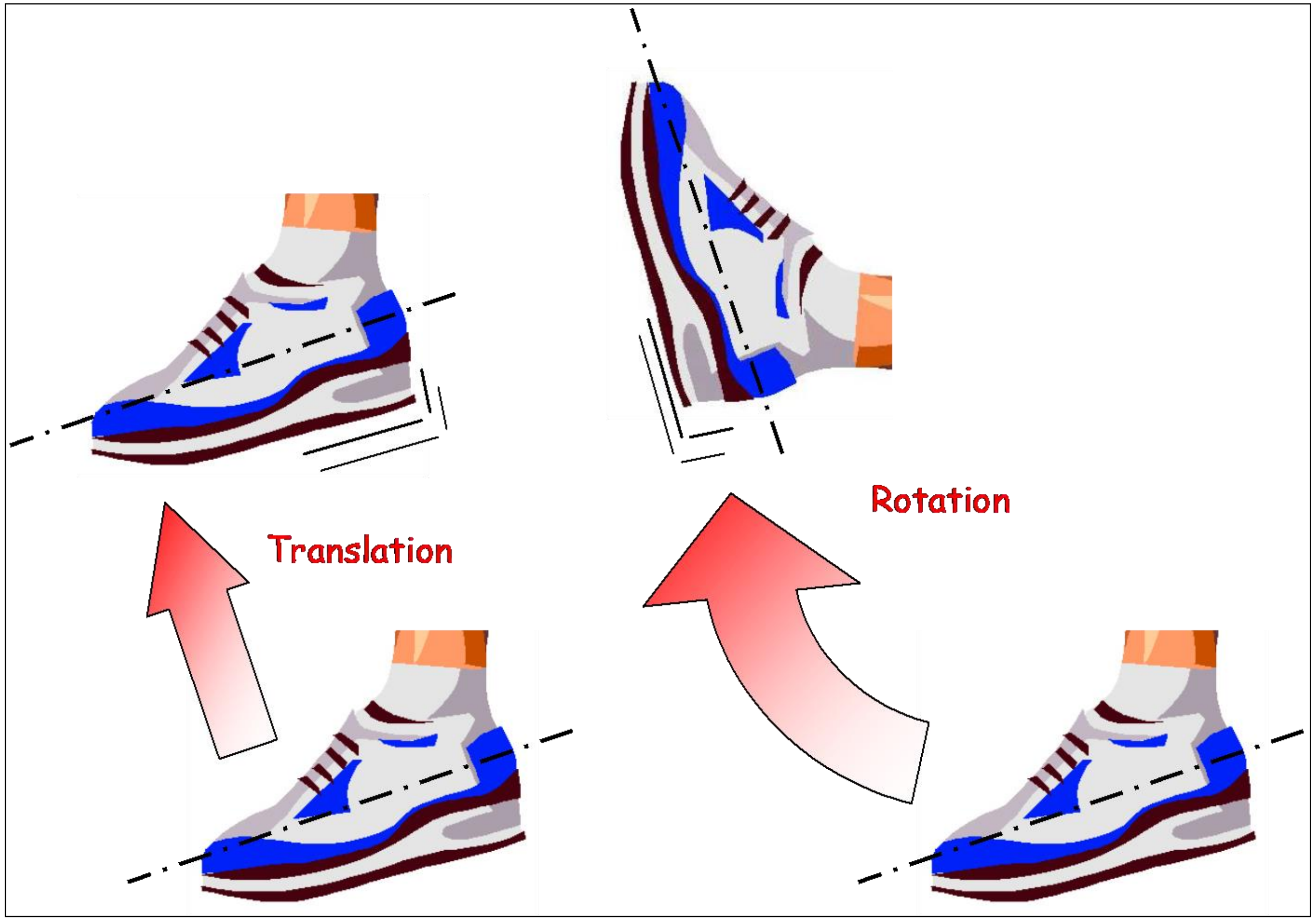
Kinematics = time-varying geometry

Coordinates



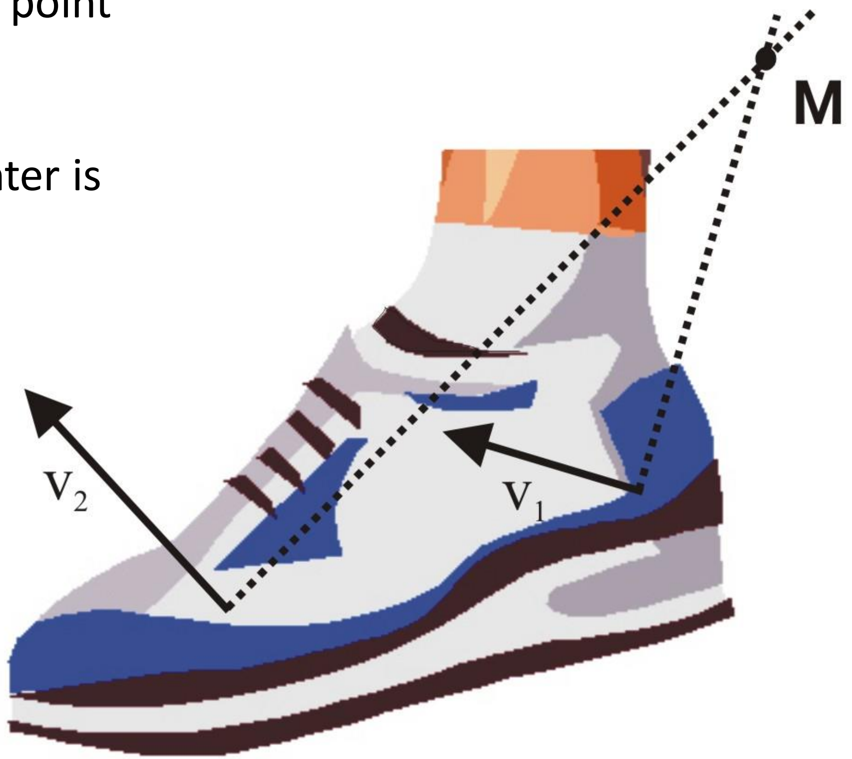
- Translationally vs. rotatory
- Absolutely vs. relative

Types of movement: translation, rotation



Instantaneous / Current axis of rotation

- Body Fixed point of the moment has no speed.
- The body rotates instantaneously around this point (about the axis).
- In a pure translation of the instantaneous center is located at infinity.



Example of use for instantaneous center

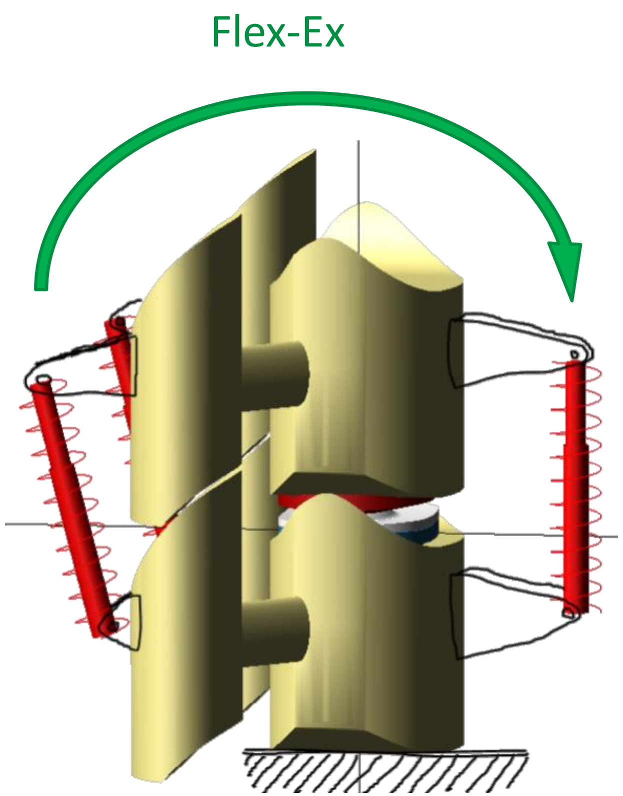
Kinematic MBS model of C5-C6

Vertebral segment with spinal disc implant

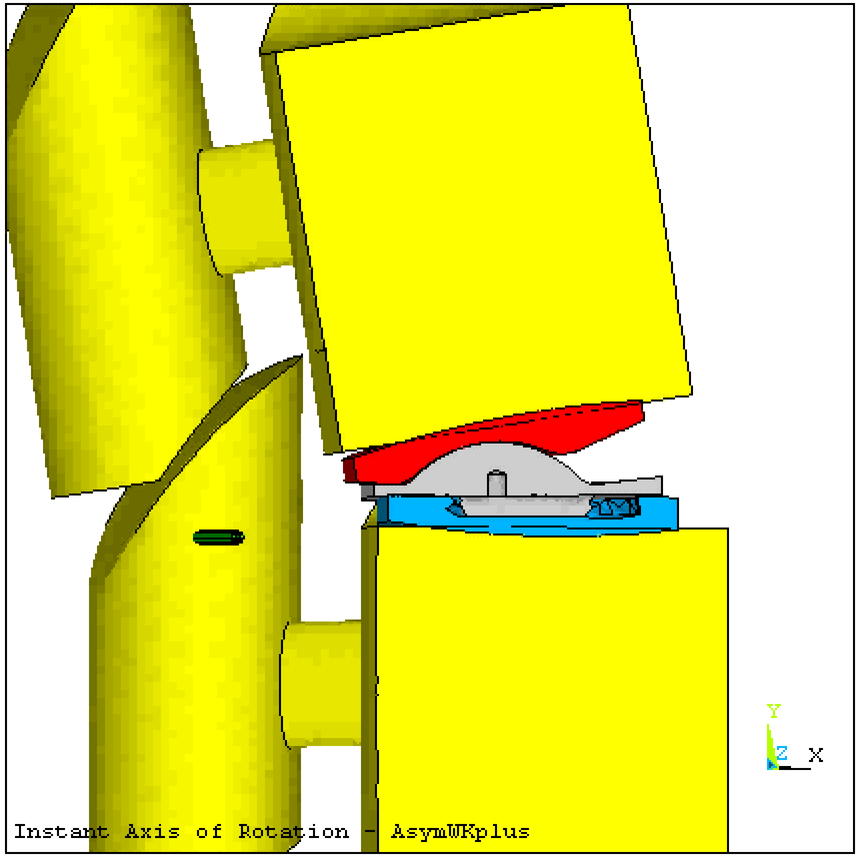
- 3D, idealized geometry
- Intervertebral disc implant
- Belts with tensile forces
- forced Flex-Ex-motion

→ **Calculating the instantaneous axis of rotation**

→ **Aim:** Implant is possible
physiological kinematics show
So for example, the unbalanced situation
the instantaneous axis of rotation below
allow the disc center.



Example of use for instantaneous center



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MAR 18 2011
07:21:01
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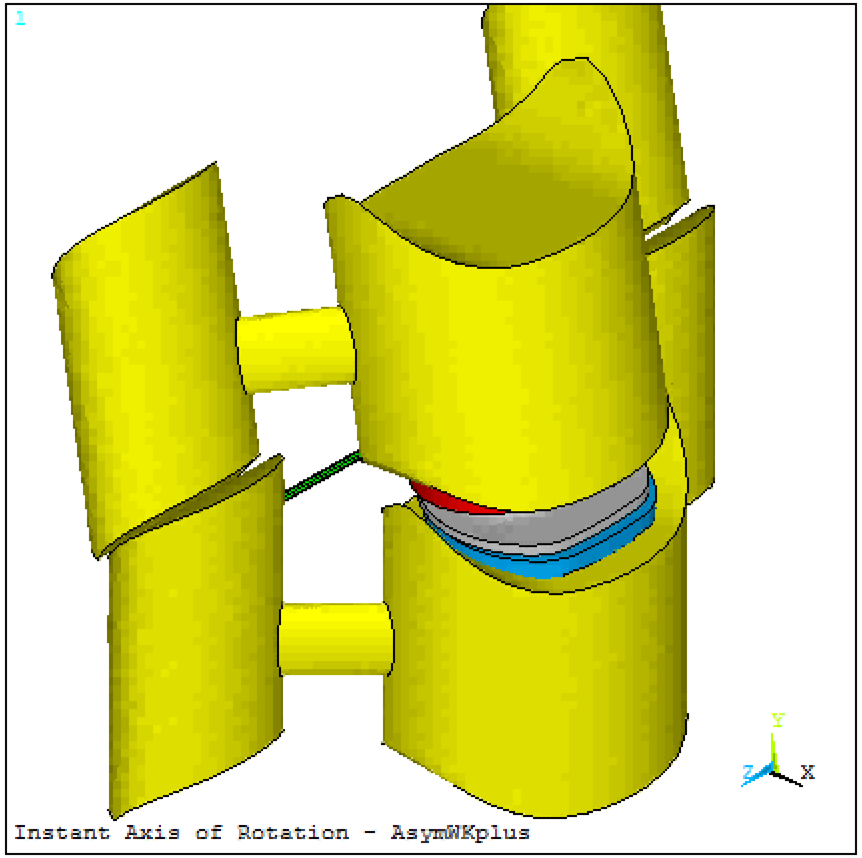
SUB =1
TIME=1.03
PowerGraphics
EFACET=1
AVRES=Mat
DMX =15.3965

Example of use for instantaneous center



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MAR 18 2011
11:44:34
DISPLACEMENT
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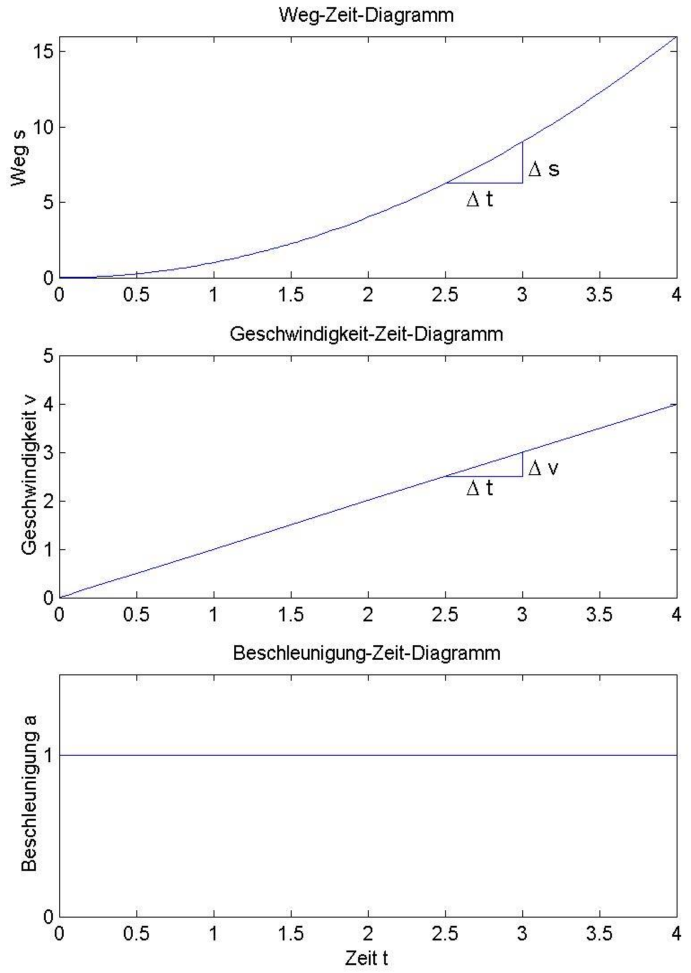
SUB =1
TIME=1.03
PowerGraphics
EFACET=1
AVRES=Mat
DMX =15.3965

Distance, speed, acceleration

Translation	**Path:** distance between <u>two</u> Points.	x	m
	Speed: The change of the way in time.	$v = \dot{x}$	$\frac{\text{m}}{\text{sec}}$
	Acceleration: The change of velocity with time (magnitude and / or direction).	$a = \dot{v}$	$\frac{\text{m}}{\text{sec}^2}$
rotation	**Angle:** inclination between <u>two</u> Axes.	φ	Grad
	Angular velocity: The change of angle with time.	$\omega = \dot{\varphi}$	$\frac{\text{Grad}}{\text{sec}}$
	Angular acceleration: The change of anglespeed with time.	$\alpha = \dot{\omega}$	$\frac{\text{Grad}}{\text{sec}^2}$



Dynamics

- Interaction between movement and forces.
- *damping, friction . inertial forces,*

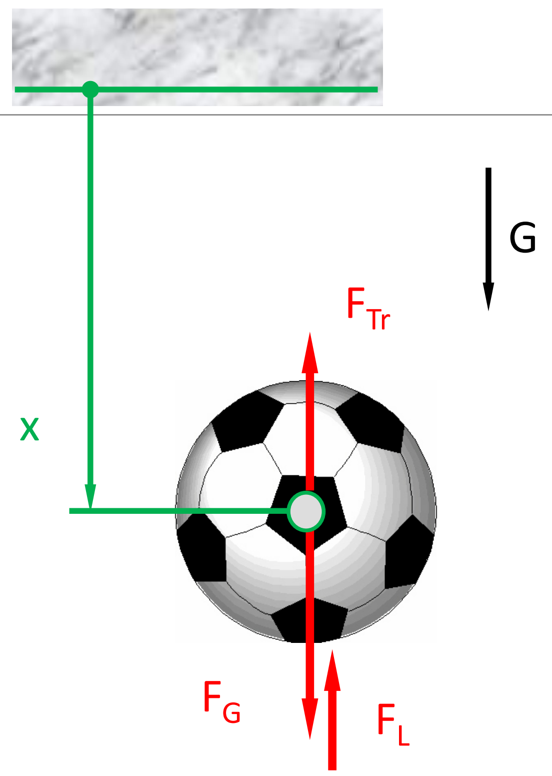
d'Alembert principle:

- Inertial forces and moments just like other external forces and moments treat. Enter the FKB.
- **dynamic equilibrium** just use as static equilibrium.

$$\sum F_{i,x} = 0$$

$$-F_{Tr} - F_L + F_G = 0$$

$$m\ddot{x} - F_L + mg = 0 \qquad \Rightarrow \ddot{x} = g - \frac{F_L}{m}(\dot{x})$$



Example: "Falling Football"
with force of gravity, air resistance
was powerful and inertial force

Energy E

unit: Joule

$$J = N \cdot m$$

Kinetic energy:

$$E_{kin} = \frac{1}{2} \cdot m \cdot v^2$$

Potential energy:

$$E_{pot} = m \cdot g \cdot h$$

potential energy

$$E_{pot} = \frac{1}{2} \cdot k \cdot x^2$$

spring energy

to Remember:

Energy is conserved.

Work W

- changes the energy content of systems.
- Forces can perform mechanical work when the force application point moves in the direction of the force.
- At constant force then:

Remember to:
Work = force times distance

unit (Such as energy): Joule $$\mathrm{J} = \mathrm{N} \cdot \mathrm{m}$$

Example lifting work: $$W_{Hub} = F_G \cdot h$$

Example frictional work: $$W_{\mathrm{Re}ib} = -F_R \cdot s$$

Power P

to Remember:

Power = energy per second

,

unit: Watt

$$W = \frac{J}{\sec} = \frac{N \cdot m}{\sec}$$

Literature

For technical mechanics:

Dankert, H. and Dankert, J."Engineering Mechanics - computer assistance".

Very good textbook

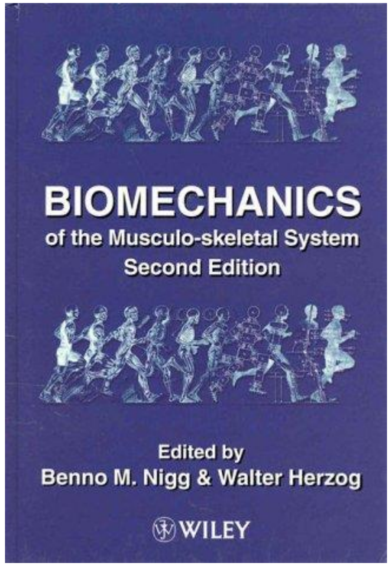
Kessel, S. and Fröhling, D."Engineering Mechanics / Technical Mechanics"

German-English technical terms in context.

The kinetics and kinematics of the musculoskeletal system:

Nigg, BM and Herzog, W."Biomechanics of the musculo-skeletal system"

Well, focus: Measurement and Modeling of the aisle.

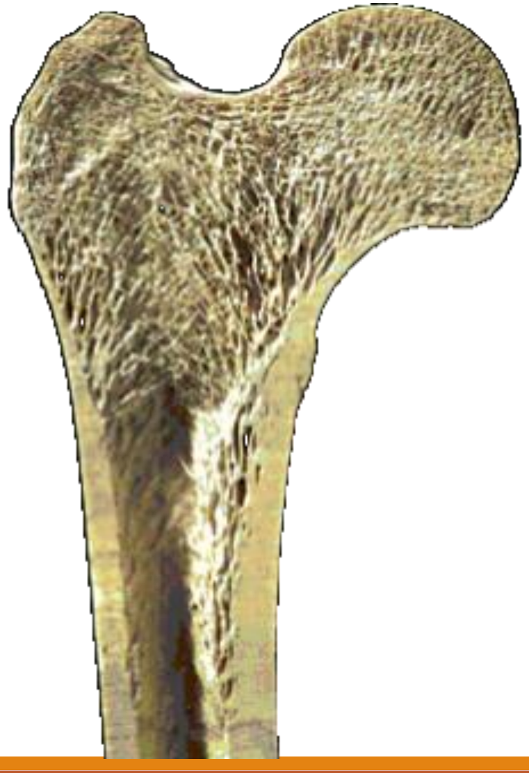


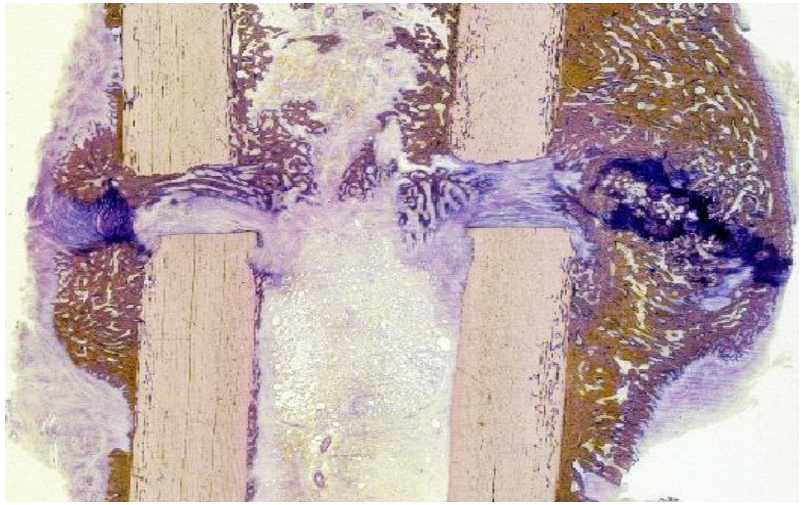
Biomechanical Principles

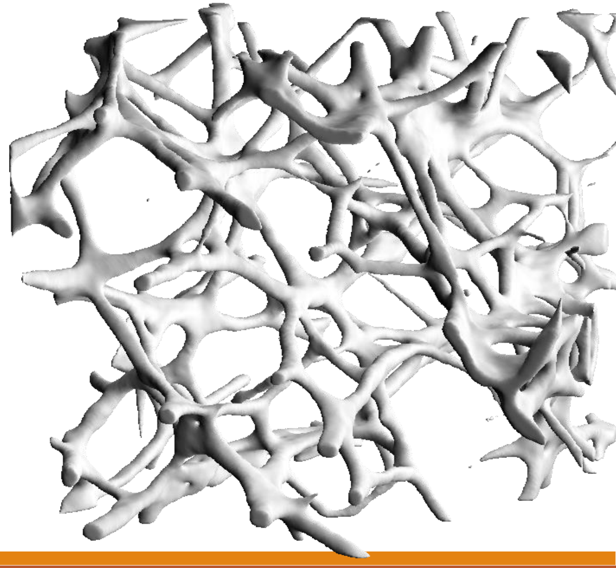
The Bone Structure

M.Sc. Lucas Engelhardt

UZWR







The overriding principle:

Roux (1895) and Wolff (1892):
"Functional adaptation"

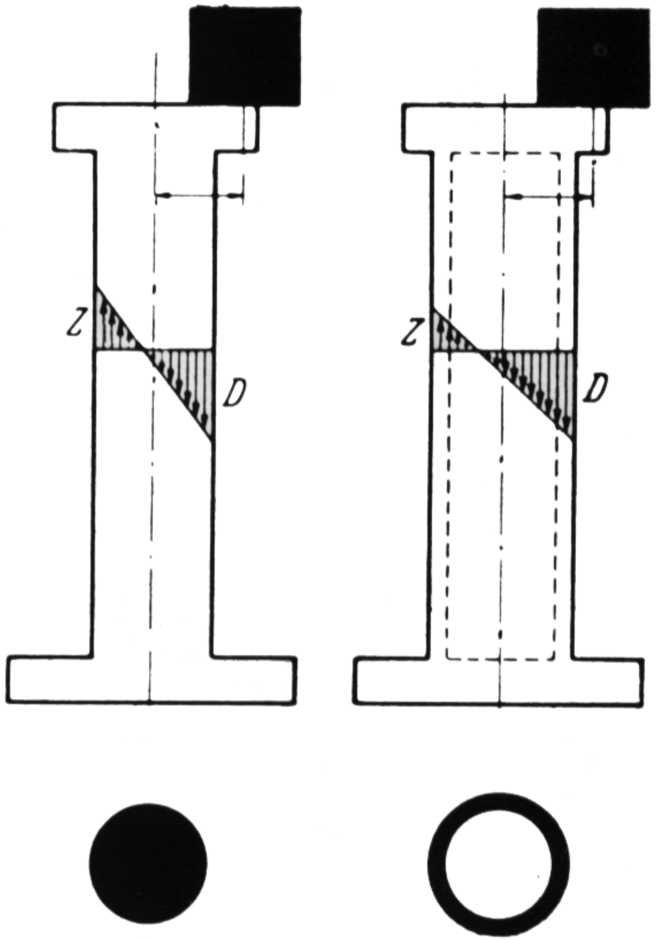
Pauwels (1965):

"Minimum-maximum principle"

With minimal amount of material (energy) a achieve maximum rigidity and strength

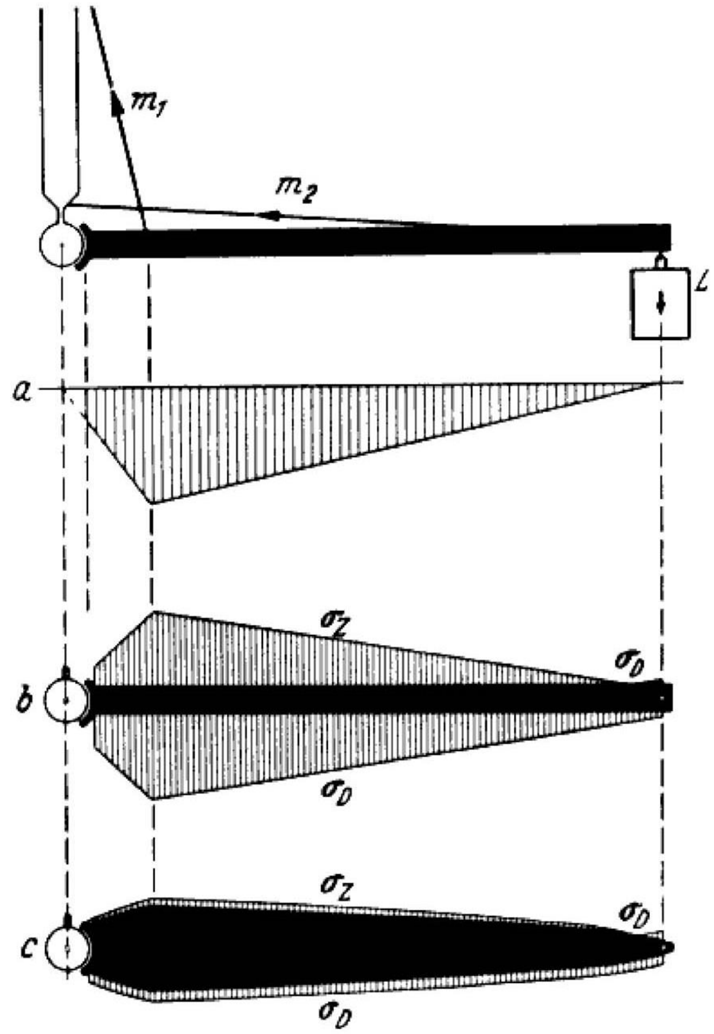
Principle: Bones

➔
In bending and torsion tube (ie same mass) stiffer and stronger than a full circle with the same surface area. See.axial and polar moments of area

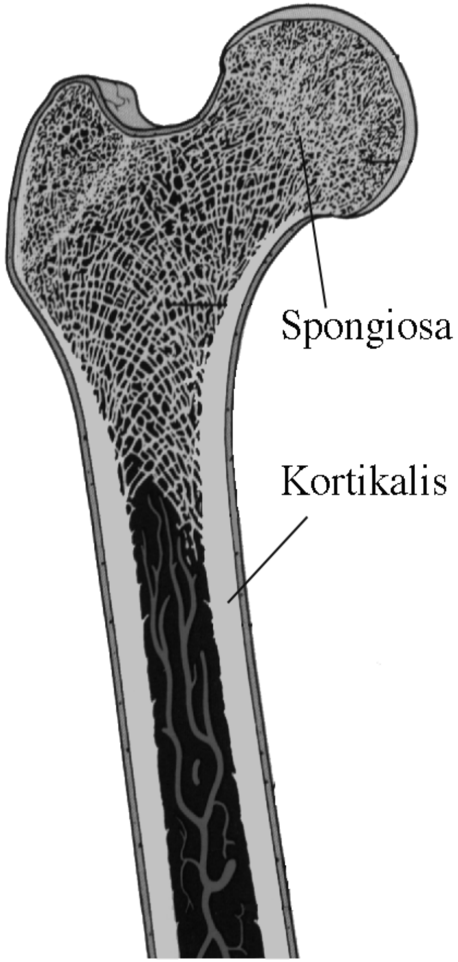


Principle: Adapted cross-sectional profile

➔
Bone cross-section adapted to bending moment.

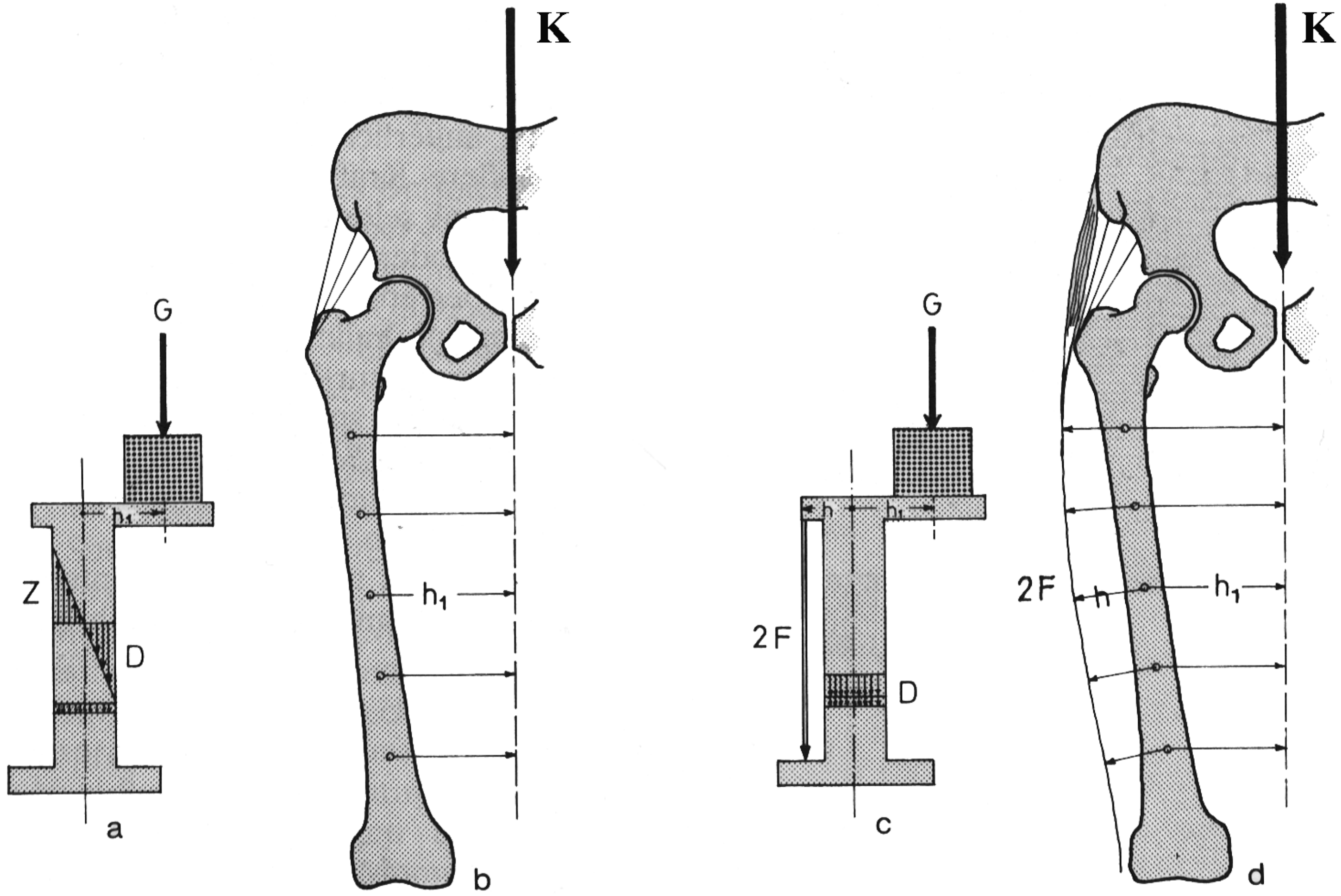


Principle: Cancellous bone



➔ cartilage requires large area. epiphyseal have larger diameter than diaphysis. Compact bone would be waste of material here. See. Lightweight principle "sandwich board".

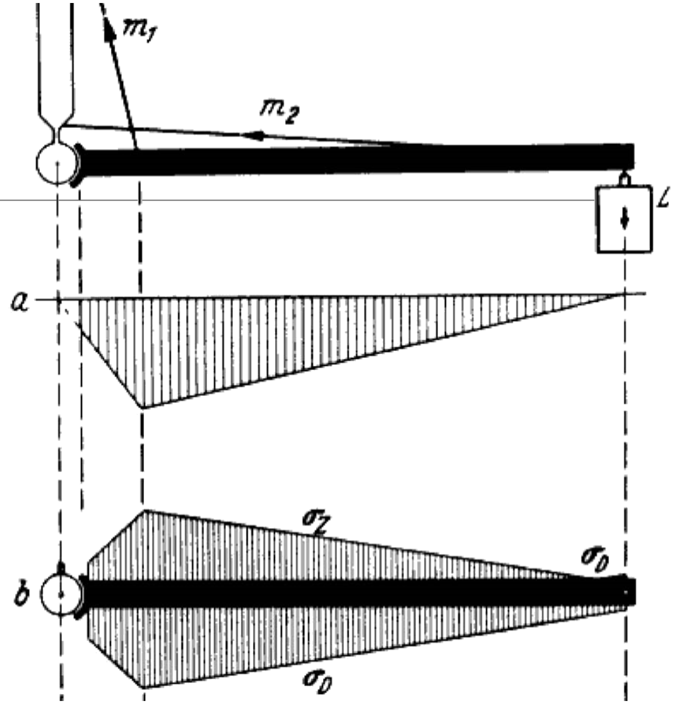
Principle: Tension band

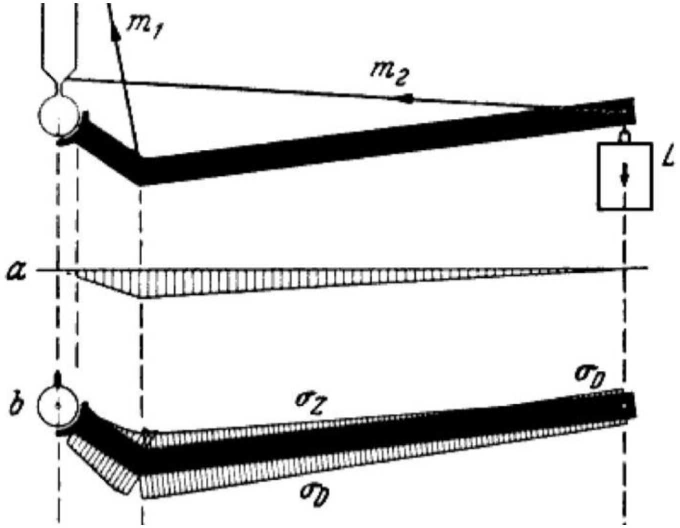


➔ bones "like" no tensile stresses! Train is therefore partially (tapestract illio-tibial) accepted. See. Prestressed concrete.

Principle: stem curvature

➔
Bone axis is partially tilted so that bone is mainly loaded in compression, rather than bending. As a result, especially the tensile stresses are reduced.





Questions and Feedback