Praktikum Simulationssoftware (SiSo)

Einführung zu P02:

FEM, Verifikation, Validierung

Ulrich Simon, Frank Niemeyer, Martin Pietsch

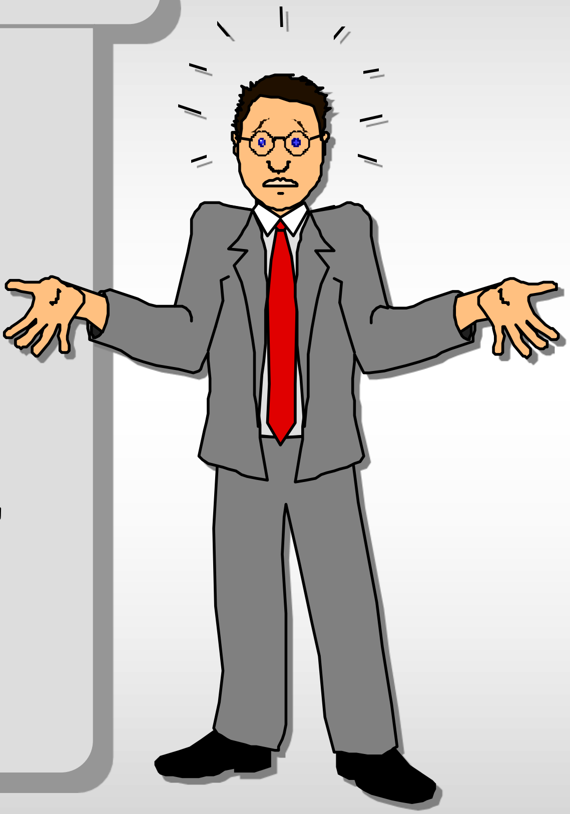
Ulmer Zentrum für Wissenschaftliches Rechnen (UZWR)
www.uni-ulm.de/uzwr

FEM: Erklärung in einem Satz

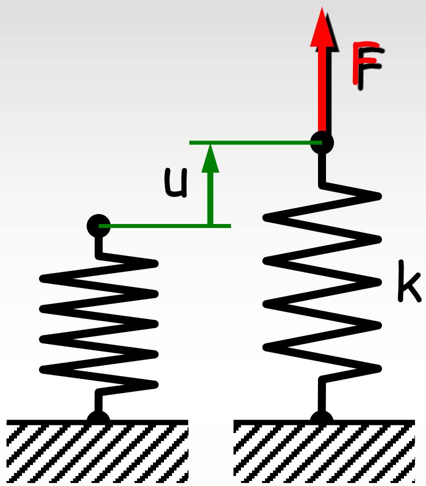
Finite-Elemente-Methode

=

„Numerisches Verfahren
zur näherungsweisen Lösung von
partiellen Differentialgleichungen"

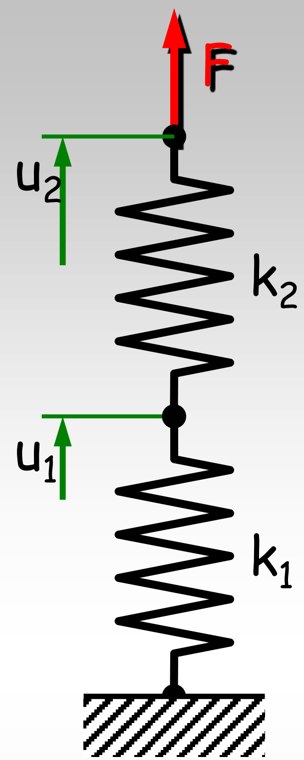


FEM: Erklärung auf einer Folie



$$k \cdot u = F$$

$$u = F/k$$



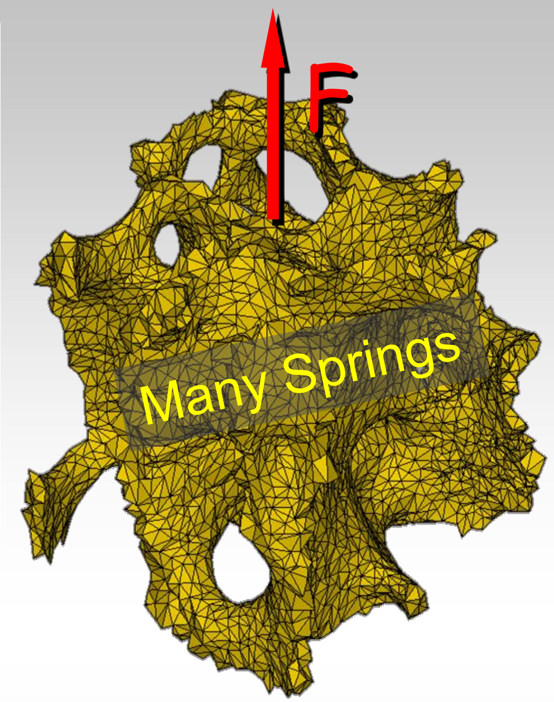
$$k_1 u_1 = k_2 (u_2 - u_1)$$

$$k_2 (u_2 - u_1) = F$$

$$\underbrace{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}}_{\underline{\underline{K}}} \cdot \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\underline{u}} = \underbrace{\begin{bmatrix} 0 \\ F \end{bmatrix}}_{\underline{F}}$$

$$\underline{u} = \underline{\underline{K}}^{-1} \underline{F}$$

$\Leftrightarrow$



FE-Software

$$\underline{\underline{K}} \cdot \underline{u} = \underline{F}$$

FE-Software

$$\underline{u} = \underline{\underline{K}}^{-1} \underline{F}$$

Anwendungsgebiete

Dynamics
- Implicit: Modal analysis
- Explicit: transient time dependent (crash)

Statics, Elasticity
- Stresses, Strains

Nonlinearities:
- Contact, Friction
- Plasticity, Hardening
- Fatigue, Fracture mechanics
- Shape optimization

FEM

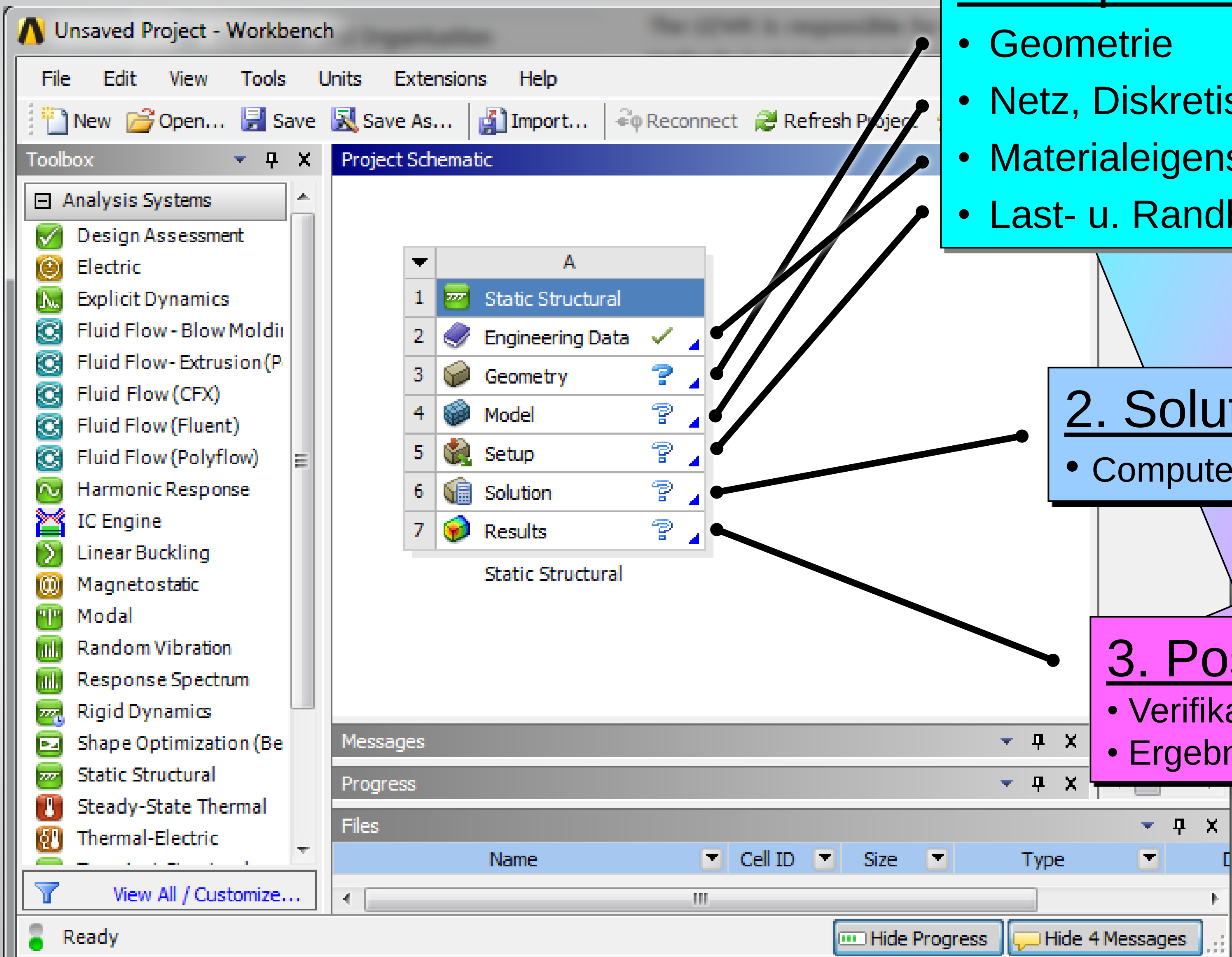
Accustics

Fluid flow
- Air planes
- Weather prediction

Heat Transfer, Diffusion

Electromagnetic fields

Arbeitsschritte



1. Preprozessor

- Geometrie
- Netz, Diskretisierung
- Materialeigenschaften
- Last- u. Randbedingungen

2. Solution

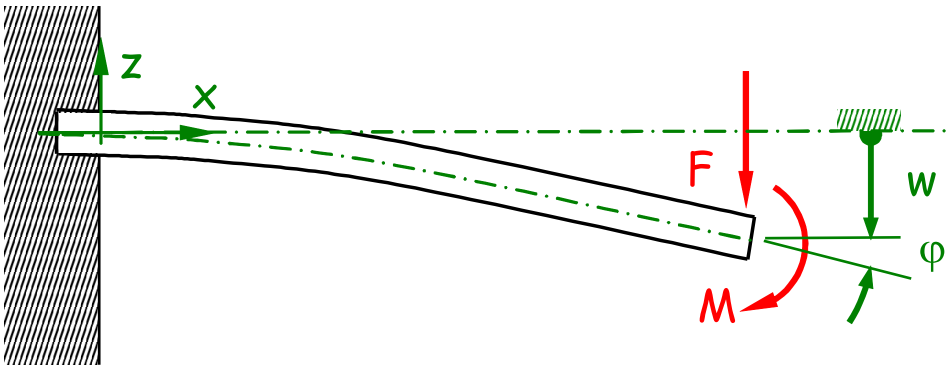
- Computer rechnet

3. Postprozessor

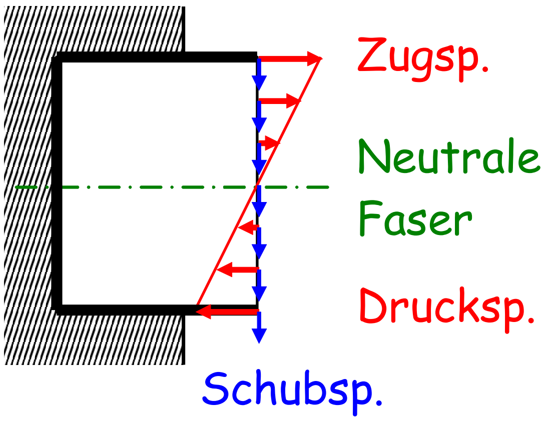
- Verifikation, Validierung
- Ergebnisse präsentieren

Balkenbiegung [Beam Bending]

Kragbalken (Biegesteifigkeit EI_a, Länge L)



Schnitt



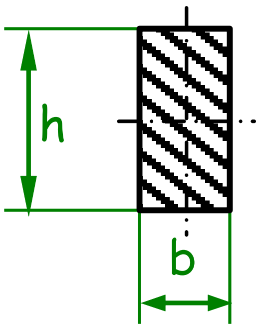
Kragbalkenformel

$$w = \frac{L^3}{3EI_a} F + \frac{L^2}{2EI_a} M$$

$$\varphi = \frac{L^2}{2EI_a} F + \frac{L}{EI_a} M$$

Flächenmomente zweiten Grades [Second Moment of Area]

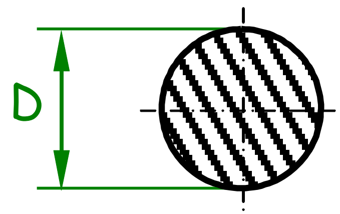
Rechteck:



Axiales Flächenmoment

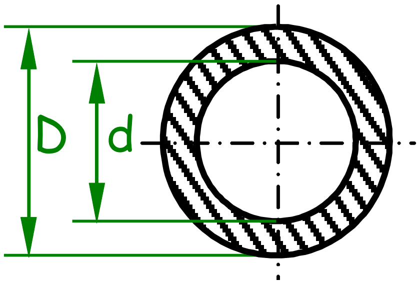
$$I_a = \frac{b \cdot h^3}{12}$$

Vollkreis:



$$I_a = \frac{\pi}{64} D^4$$

Rohr:



$$I_a = \frac{\pi}{64} (D^4 - d^4)$$